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Papers

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THE APPLICATION OF SEDIMENT-TRANSPORT MECHANICS TO STABLE-CHANNEL DESIGN

Emmett M. Laursen,[1] A.M. ASCE
(Proc. Paper 1034)

SYNOPSIS

The three requisites for a stable alluvial channel are explicitly stated and the role of sediment transport in each assessed. The similarity of the many sediment transport formulas is demonstrated and the general method of application to design illustrated. The use of these formulas as scaling relationships between different channels is advocated.

Not enough is known about the mechanics of sediment transportation for theory alone to be used with confidence in the design of an alluvial channel. However, enough is known for transport theory to be an aid to the designer in the exercise of engineering judgment. In part this aid consists of dispelling confusion by demonstrating that design methods and criteria which are seemingly different are instead basically equivalent.

By definition, a stable sediment-bearing canal in erodible material must not only convey the required water discharge, but must also transport the sediment load supplied, and must not scour or silt either the bed or the banks. These three factors neatly delineate the designer's problem, the solution to which is almost single valued—"almost" only because there may be a narrow but finite range of flow conditions above which the banks will not silt and below which they will not erode. The designer's problem is the selection of the "correct" combination of slope, size, and shape of channel which will result in a stable canal for a particular combination of water discharge and sediment load (or range thereof) and the particular materials through which the canal must pass. The selection must be based on personal experience guided by such relationships as have been deduced from general experience with existing channels and with laboratory flumes. The difficulty in selecting the

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correct combination stems from an always limited personal experience and from a lack of confidence in the guiding relationships.

In order to design a stable channel, three relationships are needed, one for each of the factors in the previous definition—a flow equation, a sediment transport equation, and a bank-erosion criterion. Although the flow conditions in the stable canal must satisfy all three of these relationships simultaneously, for clarity in exposition they will be considered separately in the following discussion.

The Flow Equation

Of the three relationships needed for the design of a stable alluvial channel, the best understood is the flow equation. Basically this relationship is the Chezy equation

$$Q = CA\sqrt{RS} \tag{1}$$

In the use of this equation the designer must assign, arbitrarily, a numerical value to the discharge coefficient C. In order to aid his judgment, he will usually prefer to use some empirical formula such as that of Manning:

$$Q = \frac{1.49}{n} A R^{2/3} S^{1/2} \tag{2}$$

This form of the flow equation was widely accepted because it was generally believed that the coefficient n was primarily if not entirely a function of the boundary material. Although it is now realized that n can be affected by several factors, a wealth of information has been accumulated over the years on the n values for existing channels covering a wide range of size, shape, alignment, bank and bed material, and condition. Because of this store of information, n values for channels similar to that being designed can usually be found. The importance of this fact is self evident, but it is noteworthy here because the same sort of aid to judgment is needed for the other two relationships.

Sediment transportation will affect the flow, and it will hence be reflected in the flow equation insofar as it affects the discharge coefficient. Dunes are almost always associated with the transport of sediment, and the resistance coefficient, which is the inverse of discharge coefficient, is affected by the resulting increase in boundary roughness. That sediment transport may influence the resistance coefficient in other ways has been claimed(1) but has not been proved unequivocally. No matter what the causal relation, however, the rate of sediment transport is one of the several criteria which should be considered in appraising the similarity of channels.

Two attempts to relate the resistance coefficient to sediment transport should be mentioned. Einstein and Barbarossa(2) divided the bed resistance into two parts, that due to the sand roughness and that due to bar, or dune, roughness. To evaluate the sand roughness they used Strickler's formula

$$n = \sqrt[6]{K_s}\,/29.3 \tag{3}$$

where K_s is the diameter (measured in feet) of the particle size coarser than 65% of the sediment. After eliminating bank effects in narrow channels, the effect of the sand roughness was subtracted from the total resistance and this remainder was related to a modification of Einstein's ψ function for the transportation of bed load.

Barton and Lin(3) have proposed a relationship between the Chezy coefficient, the Reynolds number of the flow, and the ratio between the sediment size and the hydraulic radius. Although sediment transport does not enter the relationship explicitly, the data were obtained from flows transporting sediment.

Both of these proposals are so new that there has not been sufficient time to evaluate their usefulness or their weaknesses. Both of them, however, bypass the problem of predicting dune size, which is probably the major effect of sediment transport on resistance to flow. Anderson(4) recently obtained a solution for the length of dune as a function of the Froude number and presented a limited amount of data corroborating his mathematical solution. His solution had a limit of a plane bed where the dune length became infinite. In addition he presented one example of a plane bed at a Froude number of 1.6. In measurements at the Iowa Institute, however, a plane bed (i.e., without dunes) has been obtained at a Froude number as low as 0.85. The sand in this experiment was 0.1 mm in diameter, finer than the sand of Anderson's example. Of course, it is possible for dunes to disappear because the dune height reduces to zero as well as because the dune length increases in infinity. The height of the dune, which was not and cannot be a part of Anderson's analysis, is fully as important as the length. Very likely it is more a function of the particle size and the Reynolds number of the flow than of the Froude number.

The empirical equations describing stable channels which have been developed in India(5) include relationships which are in essence evaluations of the Chezy C, just as the Manning equation is. These flow relationships naturally are subject to the same criticism as is often made of the other Indian criteria—that since they are based on data from Indian canals they are tailored to fit Indian conditions. This criticism, although valid, does not mean that due consideration should not be given to modifying the relationships to fit the conditions in other areas of the world.

Whatever flow equations and whatever numerical value of a resistance coefficient the designer chooses, he will arbitrarily, though to the best of his judgment, have placed a restriction on the physical character of the channel eventually to be selected. Before the final decision two more restrictions involving the same type of evaluation must be explicitly stated.

The Sediment-Transport Equation

If the flow entering the canal is not clear, but carries with it a sediment load, the capacity of the channel must exactly equal the rate at which sediment is supplied to the channel for regime conditions to prevail. This requirement, then, places a second restriction on the physical characteristics of the channel to be selected. Some of the many sediment-transport equations for bed load that have been proposed are listed in Table I. This table is not comprehensive nor is it intended to rate or judge the many expressions that can be found in the literature. In fact, the problem of picking a sediment-transport equation is an unhappy one that must be faced by each individual designer.

All of the equations are slightly different in form; all are based on limited data, either entirely or largely from the laboratory; each fits some data bet ter than the others; none fits all the data (which is why there are so many); none has gained the wide acceptance shared by the relatively few flow equa- tions. If unable to decide with confidence on one "best" equation, the design may use more than one, or even all, in the same manner as the hydrologist wont to do with the many flood formulas at hand.

In Table I, it should be noted that the equations suggested by Einstein(12) and Kalinske(13) in their original papers have not been used, but instead the modifications suggested in the sediment chapter of Engineering Hydraulics. In these modifications the original parameters are used, but the forms of th equations are changed to fit the data for the high rates of sediment transpor That these modified equations are not strikingly different from the other em pirical sediment-transport equations should not be surprising, because in their development the assumption is made that the velocity at the edge of th laminar sublayer v_δ (whether or not actually existent) can be substituted for the velocity at the level of the moving grain of sediment v_g . Since v_δ is, as usual, equated to $11.6 \sqrt{\tau/\rho}$ and bears little relation to v_g except that both are small and both are at levels near the bed, this is tantamount to assuming that q_s is a function of τ.

In order to compare the different transport equations more easily, they have been put into a form indicating the dependence of q_s on the mean velo ity V and the depth of flow y by neglecting the critical shear τ_c and usin the Manning equation to eliminate the slope. The similarity between not onl these but all the commonly used sediment-transport equations can thus be made apparent. One might also suspect, along with Gilbert,(14) that the ex- ponent as well as the coefficients in the transport equation should be variabl

If a sizable bed load is to be expected, the simplified forms can be used t estimate the transport characteristics of the canal. However, since it is dif ficult to measure the bed load or to predict the bed-material load entering through the headworks of proposed canal, a further modification of the form may be helpful. For example, if the first equation in Table I is written for the sediment discharge ratio, and then the ratios of the various parameters are taken for the proposed and an existing "similar" channel, relative rathe than absolute values can be estimated:

$$\frac{q_s/q\ (\text{proposed})}{q_s/q\ (\text{existing})} = \frac{B_l\,(p)}{B_l\,(e)}\ \frac{n^4\,(p)}{n^4\,(e)}\ \frac{V^3(p)}{V^3(e)}\ \frac{y^{-5/3}\ (p)}{y^{-5/3}\ (e)} \qquad (4)$$

It is interesting to note that Chezy proposed his flow equation in this ratio form,(15) and that the accumulation of n values together with descriptions o channels is essentially only a somewhat more sophisticated manner of han- dling the problem.

A further rearrangement of the simplified transport equations will put them in the form of the Kennedy equation and the variations thereof as show in Table II. The similarity between the modified transport equations and the Kennedy type of equations indicates that the physical basis for the latter equ tions is the requirement for sediment-transport capacity. That only Inglis has considered this factor explicitly is probably due to the fact that the equa- tions were derived from data of canal systems in which the sediment- discharge ratio did not vary greatly.

On the basis of personal judgment, aided to some extent by the sediment-transport equations of one form or another, the requirement for sediment capacity must be satisfied, just as the requirement for flow capacity was satisfied, and a second restriction on the combination of size, shape, and slope of channel is imposed.

The Bank-Erosion Criterion

If the banks of the canal were vertical and non-erodible, the final selection of channel characteristics satisfying only the first two requirements could be made either arbitrarily or on the basis of economics. An alluvial channel will have sloping banks of erodible material. Flow conditions along the banks, therefore, must be such that sediment will not deposit on the banks and that bank material will not be scoured. The first of these requirements will govern for canals designed for a minimum slope; the second for canals designed for minimum area. Satisfactory solution of the non-silting problem requires an understanding of sediment-transport mechanics in much greater detail than is presently available, especially of the suspended-sediment load. Knowledge concerning the non-scouring problem is somewhat more advanced, but the bank-erosion criterion is still the least satisfactory of the three relationships needed for design.

In the case of banks composed of granular material (e.g., sands and gravels) erosion will occur if the forces tending to move the particles are sufficiently large to overcome the resisting force of gravity. In the case of banks composed of cohesive material (e.g., clays) the force of cohesion will also resist movement and in fact may be much greater than the resisting force of gravity.

In associating the flow conditions in the canal to the bank erodibility the canal shape enters the problem on both sides of the relationship. The slope of the banks will obviously affect the susceptibility of the banks to erosion and the cross-sectional shape will affect the distribution of the boundary shear.

Lane and Carlson[20] have presented the relationship for the ratio of the critical tractive force on a bank slope τ_{cs} to the critical tractive force τ_{cb} as a function of the angle of the side slope and the angle of repose of the material. That is,

$$K = \frac{\tau_{cs}}{\tau_{cb}} = \cos\phi\sqrt{1 - \frac{\tan^2\phi}{\tan^2\theta}} \tag{5}$$

where ϕ is the angle of the side slope, and θ is the angle of repose. Other studies under the direction of Lane[21] have attempted to obtain the distribution of shear in a trapezoidal section. This knowledge of the effect of cross-sectional shape allows data or equations on the critical tractive force for bed material to be transferred to the banks.

The two formulas for critical tractive force most widely quoted are those of White[22]

$$\tau_{cb} = 12\ D \tag{6}$$

and of Shields(10)

$$\tau_{cb} = (S_s - 1)\ D\ \ f(11.6\ D/\delta') \qquad (7)$$

Any of the relationships for τ_c of the equations in Table I can also be used—at the discretion of the designer. It should be noted, however, that these relationships may result in a low rate of movement rather than zero movement. A criterion based on zero movement is, perhaps, unduly restrictive, because a small amount of scour can usually be tolerated and because suspended sediment will tend to add cohesive material to the banks. Application of the relationship from the first equation in Table I will result in values of permissible canal velocities approximately the same as given by Fortier and Scobey;(23) application of the relationships of White and Shields will result in smaller permissible velocities.

Very little is known concerning the resistance to movement of clays—except that some clays can stand up under very high boundary shear. One might speculate that the resistance to boundary shear should be related to the internal shearing characteristics of the clay mass in an undisturbed saturated condition. For naturally deposited material, which is not likely to be completely homogeneous, the weakest part of the mass would govern. Erosion of seams or pockets of weak material would, of course, expose the adjacent stronger material to greater forces. The only published work concerning the resistance of cohesive materials is contained in Fortier and Scobey, and even there insufficient information is given about the character of the material to serve as more than a rough guide for the designer.

This last relationship, together with the first and second, fixes the width, depth, and slope of the channel. No method is available to determine the best geometrical shape of the cross section—except examination of other channels. In fact, canals are usually dug to a shape easily handled by construction machinery, and then allowed to modify that shape by a moderate amount of scour and fill.

CONCLUSIONS

Three relationships are needed to specify a stable alluvial channel. The three relationships may be those directly stating the three requirements for the stable canal as presented herein, or they may be any three independent forms derivable therefrom. The three independent regime-theory criteria implicitly contain these same factors.

The difficulties of designing a stable canal lie not in using the general relationships, but in assigning numerical values to coefficients and even in choosing a formal equation. The essence of design procedure is shown in Fig. 1. Choice of the equation form and numerical values to describe the sediment-transport capacity will fix a relationship between the mean velocity of flow and the depth of flow as indicated by the heavy curve in the schematized diagram. Choice of the equation form and numerical values to describe the flow capacity will result in a family of curves relating velocity and depth for various canal slopes as indicated by the dashed curves. The dot-dash curve of constant tractive force describing the bank competence where it intersects the transport-capacity curve will specify one of the family of flow-capacity curves. Velocity, depth, and slope are thereby fixed, and, of course,

the width can be obtained from the continuity principle.

The interrelationship shown above constitutes the general application of sediment-transport mechanics to stable-canal design. Only a complete, comprehensive, rigorous theory, however, could substitute for a full scale model canal. It is by observation of similar canals that the designer can assign the needed numerical values, and sediment-transport mechanics can aid in assessing the degree of similarity of different canals. The parameters of the various equations can be used as comparative criteria and the equations themselves as approximate scaling relationships. If the proposed and existing conditions are treated as ratios, the discrepancies between absolute values as computed by the equations should be greatly reduced.

Practically speaking, few if any canals are completely stable. In planning remedial and maintenance work the principles of sediment-transport mechanics can be used in the same manner as in design, the principal difference being that a canal almost completely similar is then readily available for observation.

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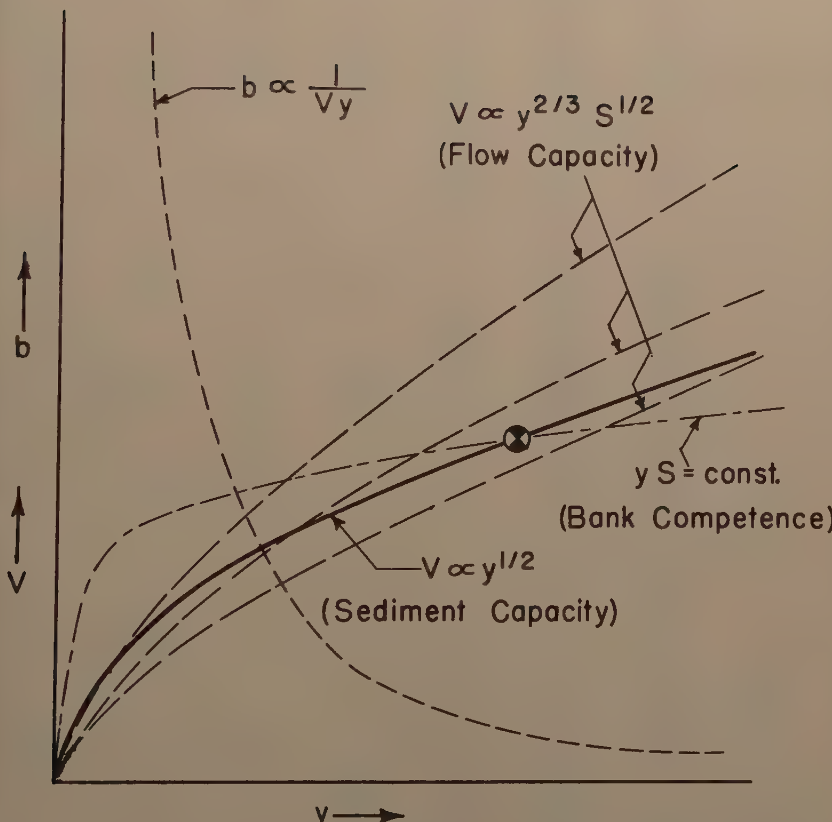

Fig. 1 Requisites for stability of an alluvial channel

Table I. Similarity of Bed-Load Formulas

DuBoys (Straub) [6]	$q_s = A_1 \, \tau (\tau - \tau_c)$	$= B_1 \, n^4 \, \dfrac{V^4}{y^{2/3}}$
Schoklitsch [7]	$q_s = \dfrac{A_2}{D^{1/2}} S^{3/2} (q - q_c)$	$= B_2 \, \dfrac{n^3}{D^{1/2}} \, \dfrac{V^4}{y}$
Meyer-Peter [8]	$q_s = (A_3 \, q^{2/3} S - A_4 D)^{3/2}$	$= B_3 \, n^3 \, \dfrac{V^4}{y}$
WES [9]	$q_s = \dfrac{A_5}{n} (\tau - \tau_c)^m$	$= B_5 \, n^{2m-1} \, \dfrac{V^{2m}}{y^{m/3}}$
Shields [10]	$q_s = \dfrac{A_6}{D} \, q \, S \, (\tau - \tau_c)$	$= B_6 \, \dfrac{n^4}{D} \, \dfrac{V^5}{y^{2/3}}$
Brown-Einstein [11]	$q_s = \dfrac{A_7}{D^{3/2}} \, \tau^3$	$= B_7 \, \dfrac{n^3}{D^{3/2}} \, \dfrac{V^6}{y}$
Brown-Kalinske [11]	$q_s = \dfrac{A_8}{D} \, \tau^{5/2}$	$= B_8 \, \dfrac{n^5}{D} \, \dfrac{V^5}{y^{5/6}}$

Table II. Similarity of Bed-Load and "Kennedy" Formulas

Duboys (Straub)	$\frac{V}{y^{5/9}} = C_1 \frac{1}{n^{4/3}} \left[\frac{q_s}{q}\right]^{1/3}$
Schoklitsch	$\frac{V}{y^{2/3}} = C_2 \frac{D^{1/6}}{n} \left[\frac{q_s}{q}\right]^{1/3}$
Meyer-Peter	$\frac{V}{y^{2/3}} = C_3 \frac{1}{n} \left[\frac{q_s}{q}\right]^{1/3}$
WES	$\frac{V}{y^{m'}}{}^{*} = C_5 \frac{1}{n} \left[\frac{q_s}{q}\right]^{1/(2m-1)}$
Shields	$\frac{V}{y^{5/12}} = C_6 \frac{D^{1/4}}{n} \left[\frac{q_s}{q}\right]^{1/4}$
Brown-Einstein	$\frac{V}{y^{2/5}} = C_7 \frac{D^{3/10}}{n^{3/5}} \left[\frac{q_s}{q}\right]^{1/5}$
Brown-Kalinske	$\frac{V}{y^{11/24}} = C_8 \frac{D^{1/4}}{n^{5/4}} \left[\frac{q_s}{q}\right]^{1/4}$
Kennedy [6]	$\frac{V}{y^{0.64}} = 0.84$
Lacey [17]	$\frac{V}{y^{1/2}} = 1.15\sqrt{f}$
Blench [18]	$\frac{V}{y^{1/2}} = \sqrt{b}$
Inglis [19]	$\frac{V}{y^{1/2}} = C_9 \frac{g^{1/3}\ w^{1/4}}{\nu^{1/6}} \left[\frac{q_s}{q}\right]^{1/4}$

* $m' = (m+3)/(6m-3)$

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MEASURING EVAPOTRANSPIRATION FROM ATMOSPHERIC DATA

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(Proc. Paper 1035)

SYNOPSIS

A method of evaluating evapotranspiration is presented which is based on the mass balance of water vapor in the atmosphere. The method is applied to various regions of the North American continent for the calendar year 1949 and the results are compared with hydrologic data and with an empirical method of estimating evapotranspiration.

It is shown that although accurate values of evapotranspiration can be obtained for large areas such as the entire continent, the accuracy of the method is reduced as the size of the area under consideration is decreased. The potentialities and limitations of the proposed method are evaluated.

INTRODUCTION

Evapotranspiration—the loss of water from natural surfaces by evaporation and transpiration—has resisted measurement more strongly and is consequently less well known than most hydrologic parameters. The addition of moisture to the land by precipitation and its depletion through surface runoff may be measured, but the rate at which the soil moisture supply is lost to the atmosphere under natural conditions cannot be measured directly by existing instruments.

Yet, a quantitative understanding of the process of evapotranspiration is of considerable importance to the meteorologist and the hydrologist. The meteorologist would like more information concerning the water balance of the atmosphere, as well as the turbulent exchange processes that take place in the microclimatic layer. On the other hand, the hydrologist would like to be able to maintain a balance sheet of soil moisture content to assist him in evaluating drought hazard or irrigation requirements, or to improve methods of river forecasting or reservoir management. Evapotranspiration is also of

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importance to the military engineer, for off-the-road trafficability is strongly affected by the water content of the soil.

Since instruments do not yet exist for measuring actual evapotranspiration under field conditions, scientists have turned to expressing evapotranspiration from natural regions by various crude formulae. Many such formulae have been developed. Some are based on simplified physical theory (e.g., Richardson and Montgomery(9))[3] but most are purely empirical (e.g., Meyer;(7) Lloyd(6)). Naturally, one of the greatest difficulties in devising and using such formulae is the lack of objective evapotranspiration measurements to use as control data. Tank estimates of evapotranspiration and soil moisture determinations in experimental areas provide the bulk of such data, although in many regions the mean annual evapotranspiration can be computed with reasonable accuracy from precipitation and runoff measurements.

A recent empirical formula for evapotranspiration which has received widespread attention was devised by Thornthwaite.(11) Thornthwaite developed a formula for potential evapotranspiration, which he defined as the maximum amount of water that could be lost from a region under a given set of climatic conditions. He also developed a simple water budget for estimating actual evapotranspiration from potential evapotranspiration. Thornthwaite's method generally yields estimates of evapotranspiration which are reasonable in their seasonal distribution and annual total (Carnahan and Benton).(3) However, such empirical estimates cannot be properly evaluated until accurate measurements of evapotranspiration become available for natural regions.

One possible method of determining evapotranspiration, which has not heretofore been adequately explored, involves an evaluation of the atmospheric water balance over the continents. The latter subject has been discussed by many authors, but little quantitative information was available until Holzman(5) presented his analysis of the relationship between the air mass and the hydrologic cycles. The concepts advanced in this paper were later extended and developed by Benton, Blackburn, and Snead.(1) In recent years our understanding of the atmospheric water balance has been greatly increased by the extensive use of electronic computers in meteorological research. As an outgrowth of this program, the flow of water vapor over the North American continent was analyzed and discussed by Benton and Estoque.(2)

The availability of extensive meteorological data and the ability to process this information numerically in an efficient manner, led to a growing realization that our knowledge of atmospheric vapor flow might be sufficient to allow the direct computation of evapotranspiration from a statement of the mass balance of atmospheric water, developed in a manner analogous to the familiar hydrologic equation. Possibly the first attempt to evaluate evapotranspiration in this manner was presented by Möller,(8) who used a sparse network of meteorological stations over northwest Europe. A more extensive study of evapotranspiration over the North American continent was presented by Benton and Estoque.(2)

The investigations of Benton and Estoque showed that for land areas of continental extent, estimates of evapotranspiration obtained from atmospheric data are at least as accurate as those which can be obtained from hydrologic estimates or from empirical relationships. The major objective of the

3. Numbers in parentheses refer to references at the end of the paper.

present study is to evaluate the extent to which atmospheric data can be successfully used to determine evapotranspiration from smaller areas. The atmospheric balance equation is derived in the following section, and applications of this equation to specific areas of various sizes are then presented. Computed evapotranspiration is compared with estimates obtained using Thornthwaite's empirical procedure, as well as with annual evapotranspiration totals obtained from precipitation and runoff records.

It is shown that although the procedure for evaluating evapotranspiration from atmospheric data is physically sound, practical difficulties arise. These difficulties, related to problems of statistical sampling, are comparatively unimportant when large areas and long time intervals are considered but become more critical as the size of the area and time interval is reduced. Nevertheless, the atmospheric method offers a fundamentally simple approach to the evaluation of evapotranspiration which may become of increasing practical value as meteorological observations and techniques of electronic data processing are improved.

Development of the Atmospheric Balance Equation

The conventional equation, expressing the basic relationship between evapotranspiration and other hydrologic parameters for any time interval, is the hydrologic equation:

$$P = E + R + \Delta S \qquad \text{(inches)} \qquad (1)$$

Here P, E, R, and ΔS are precipitation, evapotranspiration, net runoff, and change in storage of surface and sub-surface water all expressed in inches depth, respectively. Ground water flow into or out of the region can usually be neglected if a large area is considered. Unfortunately, equation (1) cannot be solved for E for periods less than a few years in length, since ΔS is not negligible for short time periods.

A similar mass balance equation may be developed for the atmosphere, expressing the relationship between evapotranspiration and meteorological parameters. The following expression is obtained for an arbitrary time interval and region:

$$E = F + P + \Delta N \qquad (\text{gm/cm}^2) \qquad (2)$$

Here F is the net outflow of atmospheric water vapor (F = outflow minus inflow); E is evapotranspiration; P is precipitation; and ΔN is the change in the precipitable water content of the atmosphere. Two terms, ΔN and F, require further explanation.

The change in precipitable water content, ΔN, is defined as the difference between the atmospheric water content at the end of the time period and that at the beginning. It is analogous to the change in storage term in the hydrologic equation, although fortunately the moisture holding capacity of the atmosphere is so low that it has none of the numerical importance of the latter term. Precipitable water, N, is defined as

$$N = \frac{1}{g}\int_0^{p_0} \frac{\rho_w}{\rho}\,dp \qquad (\text{gm/cm}^2) \qquad (3)$$

Here $\frac{\rho_w}{\rho}$ is the specific humidity, in gm/kg; p is the pressure in millibars which varies from zero to its surface value, p_o; and g is the acceleration of gravity, in cm/sec^2. Over an arbitrary time interval τ :

$$\Delta N = N_2 - N_1 \qquad (gm/cm^2) \qquad (4)$$

where N_2 is the precipitable water at the end of the interval, and N_1 is the precipitable water at the beginning.

The net outflow of water, F, from the arbitrary area is more difficult to evaluate. Let the vector $\underline{w}$ be the mean transfer of water vapor at an arbitrary point in the atmosphere. Then:

$$\underline{w} = \frac{1}{\tau}\int_0^{\tau} \frac{1}{g}\left(\frac{\rho_w}{\rho}\right) \underline{c}\, dt \qquad (gm/cm\text{-}mb\text{-}sec) \qquad (5)$$

Here $\underline{c}$ is the horizontal wind vector, in cm/sec, expressed as a function of time t. Note that $\underline{w}$ is the flux of vapor in gm/sec through a cross-sectional area one centimeter in width and one millibar in thickness. The mean transfer of water vapor integrated vertically through the atmosphere is:

$$\underline{W} = \int_0^{p_o} \underline{w}\, dp \qquad (gm/cm\text{-}sec) \qquad (6)$$

where p_o is now the mean surface pressure. Finally the net outflow of water vapor, F, from an arbitrary area A is given by:

$$F = \frac{\tau}{A}\oint \underline{\hat{n}} \cdot \underline{W}\, d\lambda \qquad (gm/cm^2) \qquad (7)$$

where $\underline{\hat{\eta}}$ is a vector of unit length normal to an infinitely small segment of the perimenter, $d\lambda$, and pointing out of the area; and the line integral is evaluated around the perimeter of the area. Note that the transfer of liquid water is neglected. It can be shown that the resulting error is negligible in all save extremely unusual local situations.

Substituting equation (7) into (2):

$$E = P + \Delta N + \frac{\tau}{A}\oint \underline{\hat{n}} \cdot \underline{W}\, d\lambda \qquad (gm/cm^2) \qquad (8)$$

All of the variables on the right hand side of equation (8) can be evaluated provided a rawinsonde or radiosonde network is available to provide estimates of wind velocity and humidity through the atmosphere. Such a network is operated by U. S. Weather Bureau and Armed Services personnel, and participating weather stations are scattered throughout the United States and North America. Equation (8) thus provides a physically sound method of computing evapotranspiration from natural areas. Furthermore, if it is possible to evaluate the various terms in the atmospheric balance equation,

he change in storage of moisture in the soil could also be determined by ubstituting the appropriate value for evapotranspiration in the hydrologic quation (1).

It must be pointed out that although equation (8) is almost exact, and in-olves no hazardous assumptions, other difficulties do arise. The net inflow r outflow of water vapor through the atmosphere is for most areas and time ntervals a small difference between two large numbers, which must be de-ermined with considerable accuracy if satisfactory results are to be ob-ained. This limits the size of the area and the length of the time interval hich can efficiently be considered. The following sections describe the ap-lication of equation (8) to actual data.

Description of Data

The calendar year 1949 was chosen for analysis because the necessary neteorological data were readily obtainable from the research project with hich this study was associated. Monthly evapotranspiration was evaluated sing equation (8) for each of four sub-continental areas, represented by four olygons with meteorological stations at each vertex. These polygons were hosen to correspond approximately to the following regions: Canada, the entral United States, north-central United States, and the Ohio River Basin. wenty-two radiosonde stations were utilized in the study, as listed in able 1. At each station measurements of the elevation of constant pressure urfaces and of humidity were recorded twice a day, 0300Z and 1500Z (i.e., reenwich Civil Time), at each of three levels in the atmosphere: 850, 700, nd 500 mb. The resulting data were entered on punchcards.

Geostrophic winds were used in evaluating the mean transfer of water apor, $\underline{W}$, at each station. The geostrophic wind is a theoretical wind which an be computed from the slope of constant pressure surfaces even in the ab-ence of actual wind observations. Geostrophic winds are comparatively un-ffected by local topography and by small scale turbulence since they are omputed from the average slope of the pressure surface over a large region, ften several hundred miles in width. For these and other reasons, they are enerally used in computing atmospheric transfers of energy, momentum, nd other physical quantities. In the present study, geostrophic winds were omputed graphically from maps routinely prepared by the Weather Bureau-ir Force-Navy Analysis Center in Washington, D. C. These winds were en-ered into the punch-cards along with pressure and humidity data.

Once the basic atmospheric data were entered into the punch-cards, the ollowing extensions were performed. For each observation, the eastward nd northward components of the geostrophic wind velocity were evaluated nd each was multiplied by the water vapor content. These two components f water vapor transfer were summarized and averaged for each station-onth at each level to give $\overline{\underline{w}}$ (equation (5)), and the integrated water vapor ansfer at each station, $\underline{W}$, was then evaluated graphically from the com-onent relationship:

$$\underline{W} = \int_0^{p_0} \overline{\underline{w}}\, dp = \underline{\hat{i}} \int_0^{p_0} \overline{w}_x\, dp + \underline{\hat{j}} \int_0^{p_0} \overline{w}_y\, dp \qquad \text{(gm/cm-sec)} \qquad (9)$$

TABLE 1

RADIOSONDE STATIONS USED IN THIS STUDY

Station	Sta. No.	Lat. North	Lon Wes
Miami, Fla.	01	25° 49'	80°
New Orleans, La.	02	30 00	90
Brownsville, Texas	03	25 55	97
El Paso, Texas	04	31 48	106
Little Rock, Ark.	06	34 44	92
Atlanta, Georgia	07	33 39	84
Greensboro, N. C.	08	36 05	79
Portland, Me.	10	43 39	70
Buffalo, N. Y.	11	42 56	78
St. Cloud, Minn.	13	45 35	94
Lander, Wyo.	14	42 48	108
Medford, Ore.	15	42 23	122
Santa Maria, Calif.	16	34 56	120
Port Hardy, B. C.	19	50 41	127
Edmonton, Alta.	20	53 34	113
Churchill, Man.	21	56 21	94
Stephenville, Newfoundland	23	48 32	58
Upper Frobisher, Baffin Is.	24	63 44	68
Yakutat, Alaska	25	59 31	139
McGrath, Alaska	27	62 58	155
Barrow, Alaska	28	71 20	156
Coppermine, Dist. of Mack.	29	67 47	115

where x and y are directed towards the east and north respectively. Vapor transfer above 400 mb was neglected. One possible error in this evaluation involves the layer between 850 mb (the lowest level of observation) and the surface. Previous work has shown that the maximum flux of vapor may occur in this layer.(2)

As a last step in computing the net vapor flux out of each region, the appropriate line integral of the component of W normal to the boundary of each polygon was evaluated as indicated in equation (7). A linear variation of the integrated water vapor transfer was assumed between stations on the perimeter, although such an assumption can lead locally to non-negligible errors. Finally, the remaining quantities required for the determination of evapotranspiration were computed. The precipitable water content of the atmosphere at the beginning and end of each station month was estimated graphically in a manner similar to that of vapor transfer, and the mean change in precipitable water for each region was then found by averaging weighted station values using the method developed by Thiessen.(10) Thiessen's method was also used in evaluating precipitation from station measurements. With net vapor flux, change in precipitable water, and mean precipitation determined, evapotranspiration was computed for each month for each of the four regions under consideration using the atmospheric balance equation.

As indicated previously, evapotranspiration was also estimated by two other methods. The first of these was Thornthwaite's empirical equation. Although Gilbert and Van Bavel (4) have shown that Thornthwaite's equation yields somewhat better results when daily temperatures and precipitation are used, the present study was limited for simplicity to the use of monthly data in accordance with the procedure outlined in the original reference.(11) Thirty-seven stations scattered through North America, were employed in order to get adequate coverage. Thiessen's method was again used to find the mean value of evapotranspiration from station values.

A further check on the values of the total evapotranspiration for any year may be obtained by evaluating the runoff, neglecting the change in storage, after equation (1). It is anticipated that such estimates would usually be within two inches of the real values.[4] River discharge data were compiled from U. S. Geological Survey records.(12) Since only United States records were readily available, three polygons (central United States, north-central United States, and the Ohio River Basin) were checked. This involved the examination of sixty natural watersheds located within these polygons.

Presentation of Results

North American Continent

As noted previously, the field of water vapor flux over the North American continent has been described in detail by Benton and Estoque.(2) As part of their study, the water balance of the continent was computed and measurements of evapotranspiration using atmospheric data were compared with hydrologic estimates and with precipitation. For the convenience of the reader, the monthly distribution of evapotranspiration for the continent is reproduced

4. Calculations of the change in storage for the year 1949 for ten sample stations, using a method developed by Thornthwaite,(11) indicates that two inches is usually a conservative estimate for large areas.

TABLE 2

EVALUATION OF MONTHLY EVAPOTRANSPIRATION FOR CANADA, 1949

	Net Div. of Water Vapor (inches)	Precipitation (inches)	Change in Precip. Water (inches)	Actual Evapo-transpiration (inches)
Jan.	-1.06	1.57	-0.08	0.43
Feb.	-0.45	0.92	+0.07	0.54
Mar.	-0.46	1.32	+0.09	0.95
Apr.	-0.02	1.09	+0.06	1.13
May	-0.51	1.74	+0.19	1.42
June	-0.56	2.38	+0.28	2.10
July	+0.40	2.82	+0.13	3.35
Aug.	+1.13	2.35	-0.26	3.22
Sept.	-0.82	2.52	-0.22	1.48
Oct.	-1.36	2.09	-0.06	0.67
Nov.	-0.34	1.66	-0.18	1.14
Dec.	-0.50	1.19	-0.15	0.54
Annual	-4.55	21.65	-0.13	16.97

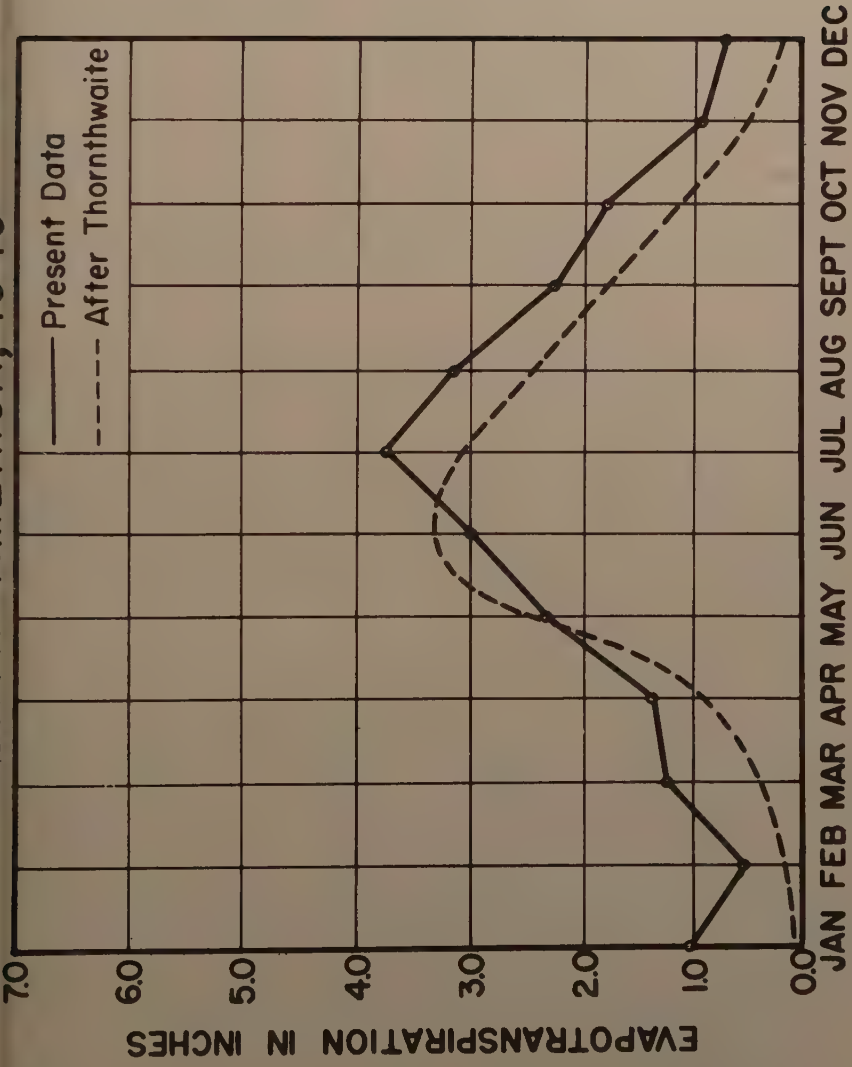


Fig. 1. The monthly distribution of evapotranspiration for the North American Continent for the year 1949 (After Benton and Estoque).(2)

in Figure 1. For a discussion, the reader is referred to the original reference.

Canada

The distribution of the monthly evapotranspiration for the area as illustrated in Figure 2 is shown in Figure 3. A sample tabulation of the various quantities in equation (8) is shown in Table 2. The relationship between the evapotranspiration evaluated from atmospheric data and that determined from Thornthwaite's empirical expression is very similar to that found for the North American continent. Values computed by use of the atmospheric balance method are higher in the winter; reach a maximum later in the year; and result in a larger annual total. These differences may in fact be caused by certain assumptions made in the empirical formula. Thus, Thornthwaite assumed that evapotranspiration ceases whenever the mean monthly temperature falls below freezing, and that potential evapotranspiration is fully realized as long as any water remains in storage in the soil. Neither of these assumptions is realistic. They would result in errors similar to the difference found in Figure 3.

Due to the unavailability of river discharge data for most of northern Canada, it is difficult to estimate the annual evapotranspiration using runoff measurements. It may be noted, however, that the annual evapotranspiration determined from atmospheric transfers of water vapor was 17.0 inches, whereas precipitation totalled 21.7 inches. This would require runoff for the year to total only 4.7 inches, if change in storage were neglected. Also, an increase in storage might be expected since precipitation in the last few months of 1949 exceeded that in corresponding months of 1948. Assuming the value of the precipitation to be correct, it becomes evident that evapotranspiration may be slightly overestimated. This apparent discrepancy and its cause will be investigated in a later section.

Central United States

The monthly distribution of evapotranspiration for this area (Figure 2) is shown in Figure 4. In general, the relationship between the curves obtained from the present method and from Thornthwaite's empirical formula remains the same, although the atmospheric balance gives consistently higher monthly values. A check on the annual total may be made since the runoff is known. For the year, precipitation amounted to 38.0 inches and runoff to 10.9 inches. Annual evapotranspiration should therefore be 27.1 inches plus a correction due to changes in storage between the beginning and end of 1949. This change was probably negative for this area primarily because November, 1949, was a very dry month (U. S. Weather Bureau).[13] A reasonable estimate of evapotranspiration might therefore be between 27 and 29 inches for the year. The computed value of 34.6 is high, and cannot be explained by inadequacies in the precipitation and runoff data. Nevertheless, the seasonal distribution of evapotranspiration is good.

It may be noted in passing that the empirical method yields an estimate of the annual evapotranspiration equal to 24.7 inches. This figure is apparently too low. The smooth curves in Figure 4 should not, therefore, be regarded as accurate. The correct evapotranspiration probably lies somewhere between the two sets of estimates.

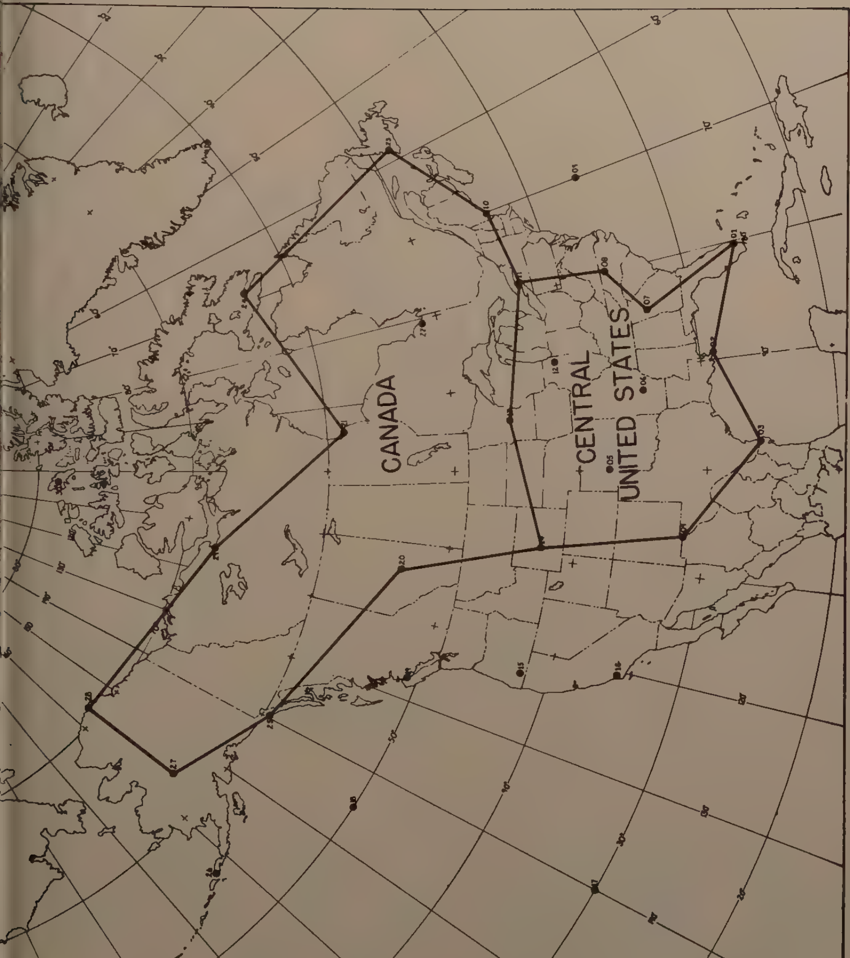


Fig. 2. Areas used to represent Canada and the central United States for evaluating evapotranspiration.

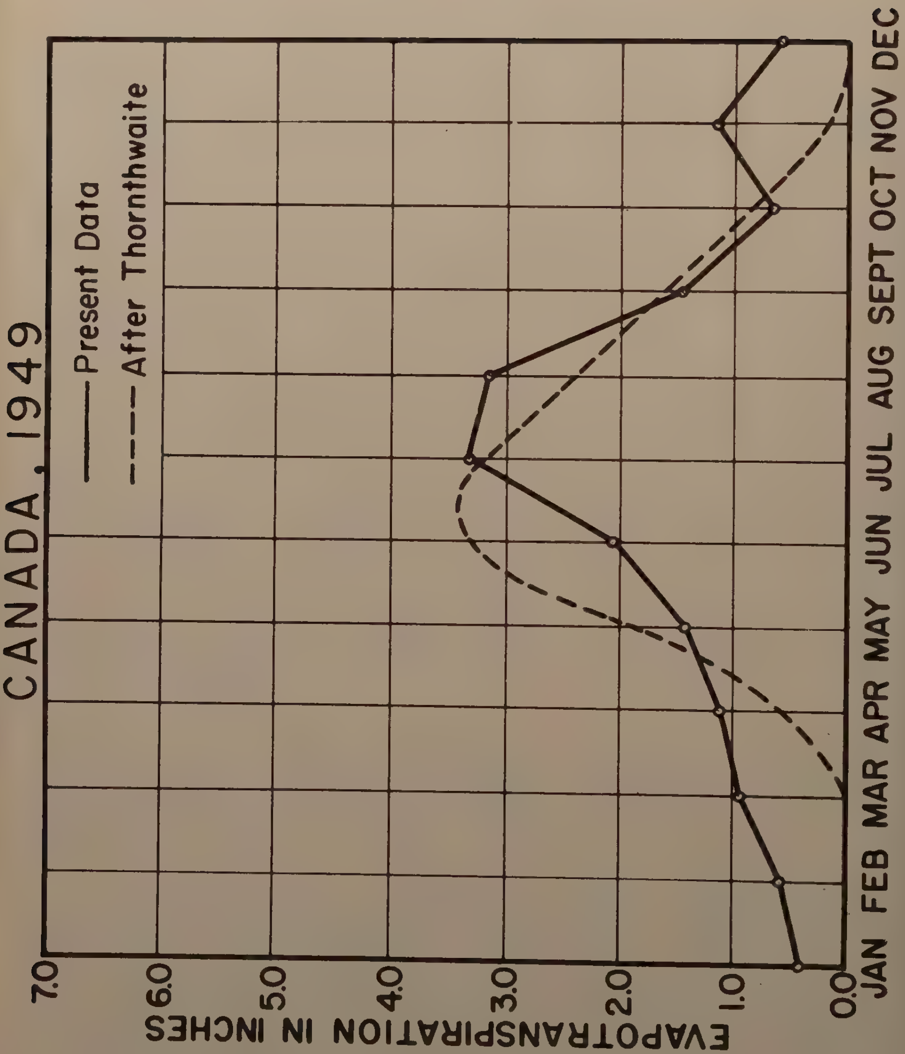


Fig. 3. The monthly distribution of evapotranspiration for Canada for

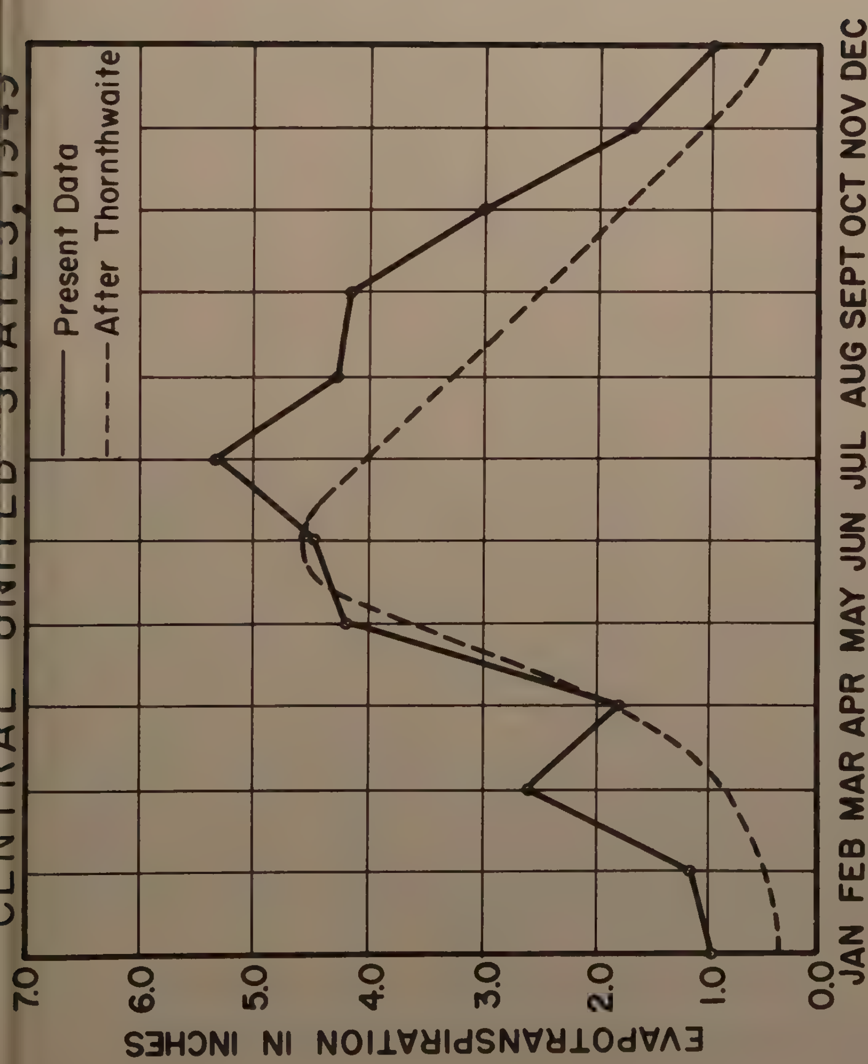


Fig. 4. The monthly distribution of evapotranspiration for the central

North-central United States

The area chosen is presented in Figure 5 and the computed monthly distribution of evapotranspiration is compared with that of the empirical method in Figure 6. Although both curves display the same general characteristics, the difference found in the examples presented above are again apparent. Thus, the empirical method yields lower values and an earlier maximum.

The annual evapotranspiration may be roughly estimated from the hydrologic equation. Precipitation for this area was found to be 44.8 inches while runoff was 16.2 inches. This results in an annual total of approximately 29 inches plus a correction due to change in storage. It is estimated that this change in storage was negative since a large portion of this polygon (the Ohio River Basin) experienced a fairly large negative change.(3) The annual evapotranspiration may, therefore, be between 29 and 31 inches. The computed value of 32.8 inches may be slightly too high; however, it is probably more accurate than the low estimate obtained by use of Thornthwaite's method (25.4 inches). It is evident that although the individual monthly values of evapotranspiration obtained from the atmospheric moisture balance may not be too precise, the seasonal distribution and the annual total are reasonably adequate.

Ohio River Basin

A polygon representing the Ohio River Basin (Figure 7) is next considered. This area was chosen primarily because much pertinent work had been done by Carnahan and Benton(3) in evaluating hydrologic parameters, including evapotranspiration, for this area and year. The comparison between the two methods (atmospheric transfer and Thornthwaite) is presented in Figure 8. The resemblance between the two sets of data is poor and the trend of the values obtained from the proposed method during the first six months seems to be opposite to what is expected. The maximum, although of the proper magnitude, occurs late in the summer season. The annual evapotranspiration, however, seems to be of the proper magnitude, 28.5 inches, compared to the "best estimate" of 28.2 inches, found by Carnahan and Benton employing many different methods. Therefore, although the monthly data and seasonal distribution are unsatisfactory the annual value seems to be about correct. Whether the discrepancies in the monthly data are caused by the fact that this is the smallest of all areas considered (826,000 km^2), will be discussed in a later section. It is interesting to note, however, that this region is not very different in location or size from the north-central United States, for which much better results were obtained. This might suggest that some systematic error, rather than a random sampling error, is involved.

Statistical Analysis of the Atmospheric Balance Equation

The atmospheric balance method as developed earlier, is based on sound physical principles and theoretically should give correct values for the rate of evapotranspiration from any selected area. From the limited number of examples furnished in the previous section, however, it is evident that certain difficulties are encountered in practice which must be removed before this method can be applied to sub-continental regions. To clarify these difficulties, statistical analysis of an idealized system has been used to determine the factors affecting the accuracy of the atmospheric balance equation.

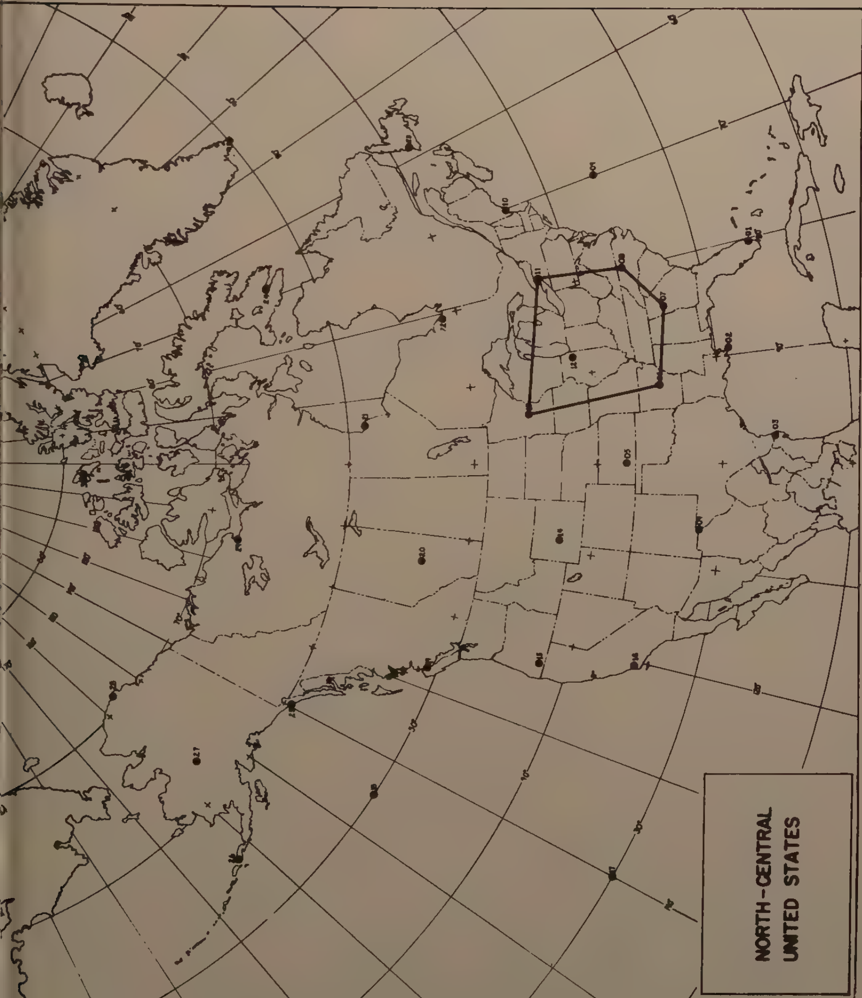


Fig. 5. Area used to represent the north-central United States for

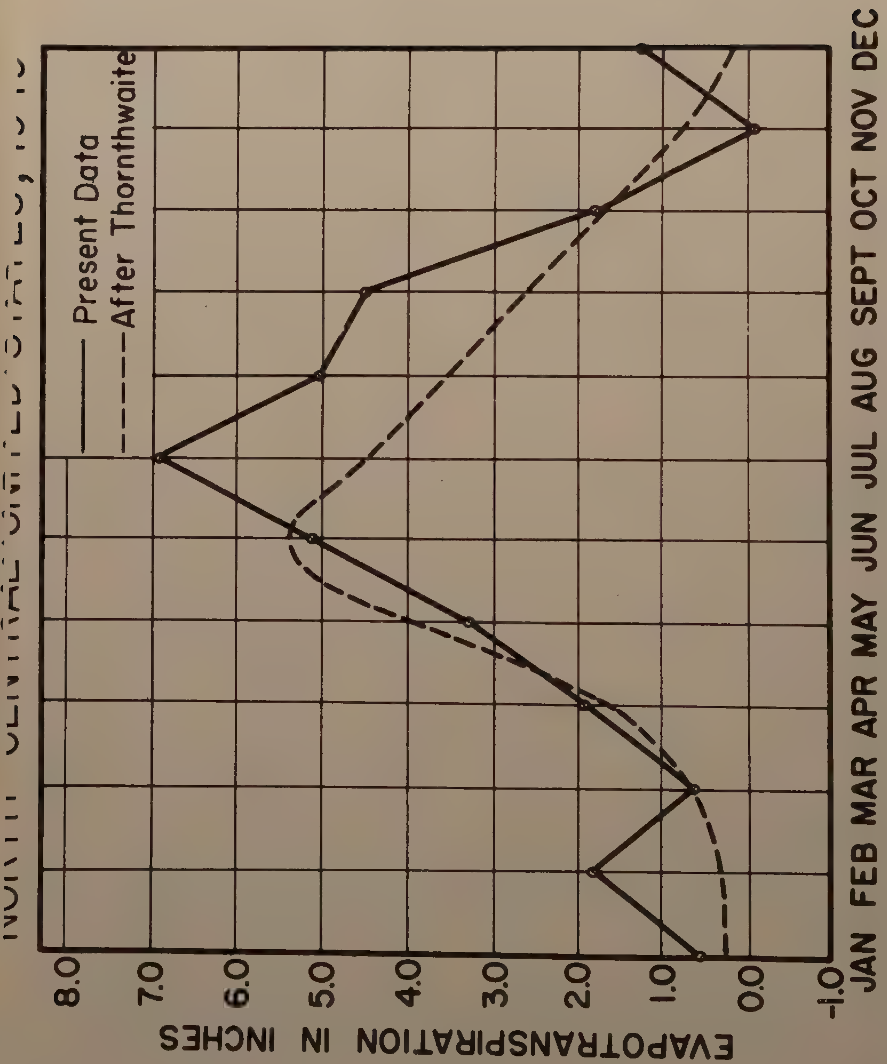


Fig. 6. The monthly distribution of evapotranspiration for the north-

Fig. 7. Area used to represent the Ohio River Basin for evaluating evapotranspiration.

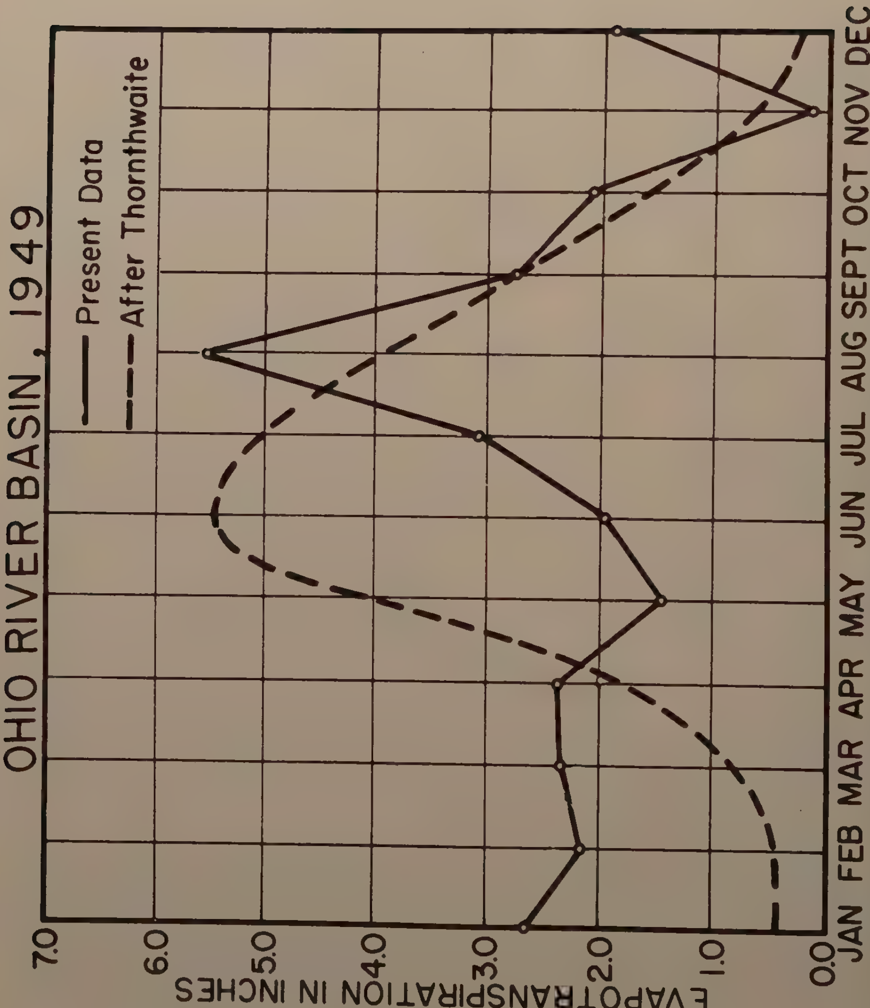


Fig. 8. The monthly distribution of evapotranspiration for the Ohio

Such analysis was limited to the divergence term since the possibility of error is greatest in evaluating the net flux of water vapor and all errors in computing the other terms are negligible in comparison.

Consider an idealized area, A, on whose perimeter, P', n stations are located with equal spacing. Each station reports N observations at η levels, each representing equal pressure increments through the atmosphere. Such a model is illustrated in Figure 9.

Let the standard error of the mean value of any component of $\underline{w}$ (the mean atmospheric transfer of water vapor at any level and station) be given by s. Integrating the mean transfer through the height of the atmosphere:

$$\underline{W} = \sum_{i=1}^{\eta} \underline{w}_i h_i \qquad \text{(gm/cm-sec)} \qquad (10)$$

where h_i is the height, in millibars, of the atmospheric layer represented by the value of the mean transfer, $\underline{w}_i$. Since all levels are spaced equally, h_i is a constant and will hereafter be denoted as h.

The standard error of estimate of any component of $\underline{W}$ (the integrated vapor transfer) is made up of two parts: error due to sampling, since only a finite number of levels are used through the atmosphere; and error due to inaccuracies in the computed values of $\underline{w}$. For averages over a moderate period of time (e.g., a week or more), $\underline{w}$ varies smoothly through the atmosphere; and hence if several levels are used the representative sampling error will be very small. Neglecting the sampling error, and assuming for simplicity that s is the same at each level, the standard error of any component of $\underline{W}$ becomes:

$$r = sh\sqrt{\eta} \qquad \text{(gm/cm-sec)} \qquad (11)$$

After converting the divergence of water vapor over an area into a line integral, and after assuming linear variation of $\underline{W}$ between stations on the perimeter:

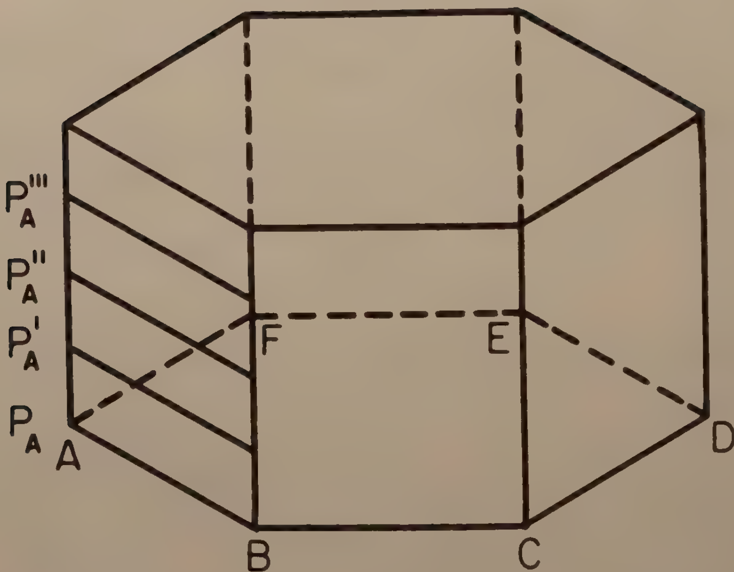
$$\nabla \cdot \underline{W} = \frac{1}{2A} \sum_{i=1}^{n} \hat{\underline{n}}_i \cdot (\underline{W}_i + \underline{W}_{i+1}) L_i \qquad \text{(gm/cm}^2\text{-sec)} \qquad (12)$$

where $\underline{W}_{n+1} \equiv \underline{W}_1$ and $\hat{\underline{n}}_i$ is a unit vector normal to the line L_i and directed outward from the polygon. Since the stations were equally spaced around the perimeter, L_i is a constant L. It will also be assumed that the standard error of estimate of each component of $\underline{W}$ does not vary from station to station. The standard error of estimate of $\nabla \cdot \underline{W}$ is then given by:

$$\text{S.E.} = \frac{rL}{A}\sqrt{\frac{n}{2}} \qquad \text{(gm/cm}^2\text{-sec)} \qquad (13)$$

Sampling error has again been neglected: i.e., the assumption of linear variation of $\underline{W}$ between stations has been assumed valid.

Introducing the perimeter P', and the number of levels η from equation



$$AB = BC = CD = \ldots\ldots$$

$$P_A - P_A' = P_A' - P_A'' = P_A'' - P_A''' = \ldots\ldots$$

Fig. 9. An idealized equilateral polygon used in evaluating errors in water vapor divergence. Vertical observations are for equal increments of pressure.

(11), the standard error in $\nabla \cdot \underline{W}$ becomes:

$$S.E. = \frac{sHP'}{A}\sqrt{\frac{1}{n\eta}} \qquad (gm/cm^2\text{-sec}) \qquad (14)$$

where H is the depth of the moisture bearing atmosphere. The 0.95 confidence interval for $\nabla \cdot \underline{W}$ is given by:

$$\nabla \cdot \underline{W} \pm \frac{sHP'}{A}\sqrt{\frac{2}{n\eta}} \qquad (gm/cm^2\text{-sec}) \qquad (15)$$

Although this formula was computed for an ideal case, and certainly not for any of the polygons treated in the text, valuable information may, nevertheless, be obtained from it. Thus it can be seen that the accuracy of the divergence term may be increased by employing a greater number of stations, levels, and observations. It is also evident that the size and shape of the area also play an important role in obtaining a true estimate of the divergence. An optimum region would be a large area whose shape approximates a circle.

Let us now consider the problem of obtaining a numerical value of the confidence interval from expression (15). The only unknown term is s, the standard error of estimate of the components of $\underline{w}$ (the mean transfer of water vapor at any station-level). It is obvious that s includes three types of error: representative sampling error, random instrumentation error, and consistent error. The last type of error is comparatively unimportant, since its percentage effect will not increase upon differentiation. Thus we may write:

$$s \approx \sqrt{s_1^2 + s_2^2} \qquad (gm/cm\text{-mb-sec}) \quad (16)$$

where s_1 is the contribution due to representative sampling error, and s_2 is the contribution due to random instrumentation error.

By estimating the average standard deviation of wind speed and mixing ratio within a twelve hour period (the interval between successive observations) it was found that a reasonable figure for s_1 was $0.7/\sqrt{N}$ gm/cm-sec. Also, if the observational error in measuring wind speed is about 1.5 m/sec and in measuring mixing ratio is about 0.3 gm/kg, s_2 will be approximately $0.8/\sqrt{N}$ gm/cm-mb-sec. Thus the value of s is about $1.1/\sqrt{N}$.

Subject to the stringent assumptions inherent in the derivation, the confidence interval for $\nabla \cdot \underline{W}$ thus becomes:[5]

$$\nabla \cdot \underline{W} \pm \frac{1.6HP'}{A}\sqrt{\frac{1}{n\eta N}} \qquad (gm/cm^2\text{-sec}) \qquad (17)$$

where N is the number of observations at each station-level.

5. For example, it must be noted that expression (17) would give a confidence interval equal to zero if an infinite number of stations, levels, or observations were employed. It is clear, therefore, that expression (17) is only approximate as a result of the idealization of the problem and must be interpreted carefully.

The Accuracy of Evapotranspiration Estimates

In the present section, the confidence interval for divergence of water vapor has been computed for four sub-continental regions and for North America using equation (17). The data are presented in Table 3. It is evident that the P/A ratio is the dominant factor. As an approximate rule, the accuracy of the method varies inversely as the square root of the area.

Table 4 compares the values of the annual evapotranspiration obtained by the proposed method with those obtained from other methods. In four out of eight comparisons the confidence interval does not include the estimate derived by use of other methods. Although this is not conclusive, since the other methods are themselves subject to considerable question, the possibility must be considered that the confidence intervals are too low. This is likely, since the assumptions made in deriving equation (17) were not always realistic.

In particular, two assumptions are open to question. First, it was assumed that there were no sampling errors introduced due to interpolations between levels or in extrapolation to the surface. However, water vapor transfers were computed only at 500, 700, and 850 mb, and as has been noted earlier the extrapolation to lower levels is very critical, since maximum transfers sometimes occur below the 850 mb level. Although every effort has been made to achieve accurate extrapolations, there is no assurance that the results are completely satisfactory.

A second assumption which may often be invalid is that the variation of water vapor transfer from one station to another can always be adequately described by a linear function. Assuming the average spacing of stations to be more or less constant, errors due to non-linearity would tend to increase as the size of area (and hence the number of stations) is decreased. It is entirely possible that errors of this type also appear in Table 4.

It is of interest to note that inaccuracies in the evaluation of the atmospheric balance equation which arise due to lack of adequate representative sampling in the vertical and around the periphery of the area can largely be eliminated by use of more levels and a larger number of radiosonde stations. The improvement in evapotranspiration estimates to be obtained by the use of more extensive meteorological data has yet to be determined by actual computation. However, it is entirely possible that the success achieved for the North American continent can be extended to areas as small as the Ohio River Basin and to time intervals as short as ten days.

CONCLUSIONS

This study has presented a method of evaluating evapotranspiration based on sound physical principles. The method has been applied to various subregions of the North American continent; and the estimates obtained have been compared with hydrologic data where available, and with Thornthwaite's empirical method. The proposed method yields reasonable monthly and annual totals of evapotranspiration when very large areas of almost continental size are considered. However, the technique may become unsatisfactory when substantially smaller areas (e.g., of the size of the Ohio River Basin) are treated.

One obvious possibility is that this decreased accuracy for smaller areas is related to statistical sampling, and the fact that the divergence of water

TABLE 3

POLYGON STATISTICS

Polygon	No. Sta.	Perimeter 10^5 cm	Area 10^{13} cm^2	P/A 10^{-8} cm^{-1}	Confidence Limits Div. of Water Vapor[a] inches Monthly	Annual
North American Continent	18	19,624	15,977	1.23	0.18	0.62
Canada	12	15,408	10,769	1.43	0.25	0.86
Central United States	9	8,919	4,005	2.23	0.45	1.56
North-central United States	5	4,462	1,269	3.52	0.95	3.29
Ohio River Basin	5	3,642	826	4.41	1.19	4.12

$N = 60$, $\eta = 4$, $H = 600$ mb, and $s = 1.1/\sqrt{N}$ gm/cm-mb-sec

[a] Note that 1 inch per month is almost equal to 100×10^{-8} gm/cm^2-sec. This convenient conversion was used in computing the confidence limits.

TABLE 4

COMPARISON OF EVAPOTRANSPIRATION DATA

Polygon	Evapotranspiration (inches)		
	Hydrologic Equation	Thornthwaite's Empirical Equ.	Atmospheric Balance Equ.
North American Continent	--	16.5	22.1 ± 0.62
Canada	--	14.1	17.0 ± 0.86
Central United States	27-29	24.9	34.6 ± 1.56
North-central United States	29-31	25.4	32.8 ± 3.29
Ohio River Basin	28	27.4	28.5 ± 4.12

vapor represents a small difference between two large numbers (the total inflow and outflow). Statistical analysis of an idealized system leads to a formula for the confidence interval for evapotranspiration which verifies that the error should increase as the area is decreased. However, numerical evaluation indicates that for smaller areas this estimate of the confidence interval is probably too small. This may, of course, be due to inadequacies in the expression for the confidence interval. For example, it was assumed that errors in extropolating moisture transfer from 850 mb to the surface and in assuming linear variation of transfer between stations were negligible.

Despite the difficulties encountered, the potentialities of this method cannot be discounted. Improved accuracy can surely be obtained by use of more meteorological stations and more levels of observations. The use of observed winds for smaller areas is also a possibility, although the adequacy of moisture divergence fields obtained from such transfers is open to question. In any case, the atmospheric balance method has yielded monthly estimates of evapotranspiration for large areas which are at least as accurate as those obtained from any other known method. With further refinements of meteorological observations and techniques of data processing, this method of measuring evapotranspiration may prove to be of considerable practical interest to the engineer.

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PRINCIPLES OF FLOCCULATION RELATED TO WATER TREATMENT*

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SYNOPSIS

Chemical coagulation is a basic water treatment process, which prepares the water for subsequent treatment operations. Coagulation includes addition of chemicals, their distribution, floc formation and floc build-up. The conditioning process that leads to floc formation and build-up is "flocculation." Flocculation is usually accomplished by prolonged agitation at low velocity.

Commonly used coagulants are alum and ferric sulfate. Alum's ionization, hydrolysis, and reaction with alkalinity are of interest but the process of coagulation cannot be explained on the basis of simple chemical relationships. Colloidal particles play an important role in coagulation. Some of the significant characteristics of colloidal dispersions are large specific surface area, high adsorptive capacity, and particle charge. Colloidal particles commonly encountered in natural waters carry a negative charge, whereas those formed during the initial phase of the coagulation process often have a plus charge. Ions possessing a charge opposite to those borne by the colloidal particles tend to precipitate the latter. An ion's precipitating power increases tremendously with its valence.

Important factors in the coagulation process are pH, dissolved solids, temperature, time-concentration, coagulant aids, exchange capacity, and mixing. The last named is essential for efficient coagulation and consists of two stages: rapid mixing for a short period followed by slow mixing for a much longer time. Flocculation is accomplished in the latter mixing stage.

Laboratory studies of the coagulation process are essential. Flexibility is an important aspect of plant design.

INTRODUCTION

Clarity is one of several highly-regarded characteristics of water. This

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is especially true if the water is to be used for municipal purposes. Some industries also require process water virtually free of turbidity. The treatment of waste water—sewage and industrial wastes—involves concentration and separation of suspended matter, that is clarification of the waste water.

One of the oldest methods of water treatment is the removal of suspended solids by subsidence.(1) However, it is a well known fact that the nature of the suspended matter in most natural waters is such that subsidence, unassisted, will not, even in combination with rapid sand filtration, render the water suitable for general use. Chemical coagulation, a water treatment keystone, is an almost universal requirement. Coagulation prepares the water for sedimentation and greatly increases the efficiency of the latter. Coagulation plus sedimentation prepare the water for filtration. Coagulation ranks high on the list of things that must be done to render water safe, clear and palatable.

Much has been learned about the whole process of coagulation especially in the last 25 to 30 years; yet, today the practice of coagulation has certain of the empirical characteristics of an art. The process is complex; it is sensitive; and natural waters are endowed with rugged individuality. We are probably just beginning to obtain a fairly clear picture of a process that has been practiced for centuries. There is still room for exploration.

Coagulation - Flocculation

The coagulation process includes "addition of chemicals to the water, their uniform distribution through the water, and the building up of a readily settleable floc, usually by prolonged agitation".(2) The process of conditioning that leads to floc formation and build-up is termed "flocculation," the subject of this discussion. Obviously it is difficult to separate "flocculation" from the overall process of "coagulation." Any discussion of the former cannot help but include the latter. Therefore much of this discussion relates to the process of coagulation which includes flocculation.

The aim of flocculation is the promotion of coalescence of the precipitates derived from the coagulating chemicals and the natural suspended matter into fast settling precipitates termed "flocs" in water treatment parlance. As will be mentioned subsequently in this paper "conditioning" or "promotion-of-coalescence" is commonly accomplished by slow mixing by hydraulic or, more frequently, mechanical means. An important step in the coagulation process, flocculation appears to merit the attention it has received especially in recent years.

HISTORICAL NOTES

Mankind because of his need for water and his curiosity regarding it has, through the ages, exhibited much interest in its qualities and the means by which they might be altered the better to serve his current needs. Consequently it is not surprising to note that coagulation of water was practiced in ancient times. Apparently there was appreciation of the need for mixing as early as the sixteenth century for an Italian physician who visited Egypt then described the use of almonds for coagulation as follows:

". . . they smeared the edges of the vessel with five sweet almonds, properly crushed, and grasping the almonds in the hand suddenly plunged hand

and arm into the water up to the elbow and moved elbow and hand vigorously and violently this way and that through the water stirring it up until they had made it far more turbid than before. Then they withdrew the arm from the vessel leaving the almonds in the water. They let it clarify and it was properly clear in three hours time."(1)

In the United States coagulation was used in connection with a New Jersey public water supply in 1885 and studies of aluminum sulfate as a coagulant were undertaken at Rutgers University during the same year.(1) Since 1885 coagulation "has been, with few exceptions, part and parcel of rapid filtration in America."(1)

Some Chemical Aspects of Coagulation

Chemicals(3)(17)

Numerous chemicals have been considered for use as coagulants. However, relatively few are in common use today. These are listed in the following table.

TABLE I

CHEMICAL COAGULANTS

Name	Approximate Formula
Aluminum Sulfate (Alum, Filter Alum)	$Al_2(SO_4)_3\ 15H_2O$
Ferric Sulfate (Ferrifloc, Ferrisul)	$Fe_2(SO_4)_3 9H_2O$
Ferric Chloride	$FeCl_3$ or $FeCl_3 6H_2O$
Chlorinated Copperas	$FeCl_3$ and $Fe_2(SO_4)_3$
Ferrous Sulfate (Copperas)	$FeSO_4 7H_2O$
Lime	
Hydrated lime	$Ca(OH)_2$
Quicklime	CaO
Activated Silica	SiO_2

An important characteristic of aluminum and iron salts is their ability, under appropriate conditions, to form gelatinous-appearing precipitates or "flocs." A more precise term would be "chemical flocs" to distinguish them from "biological flocs" produced in sewage treatment processes. Of the chemicals listed in Table I aluminum sulfate (alum) is probably the most widely used. It is cheap and generally available. Lime is frequently applied merely as a coagulant aid but in some waters it alone can produce effective coagulation. Lime is a widely used softening and coagulating agent for the hard waters encountered throughout much of the mid-western section of the United States. Consequently its inclusion in the list of commonly used coagulants is believed justified. Activated silica is a newcomer and is generally

considered a coagulant aid. However, at least one authority(3) has classed it as a coagulant.

Chemical Reactions

Although alum has certain inherent chemical peculiarities it is widely used and therefore alum has been selected to illustrate certain coagulation reactions. The commercial product is approximately $Al_2(SO_4)_3\ 15H_2O$. (Detailed specifications have been prepared by the American Water Works Association).(4) When alum is dissolved in water it ionizes yielding aluminum ions (Al^{+++}) bearing a high positive charge and negatively charged (SO_4^{--}) ions. This may be illustrated by the equation

$$Al_2(SO_4)_3 \rightarrow 2Al^{+++} + 3SO_4^{--}$$

Alum also hydrolyzes readily as follows:

$$Al_2(SO_4)_3 + 6H_2O \rightarrow 2Al(OH)_3 + 3H_2SO_4$$

In most natural waters the sulfuric acid formed by this reaction is promptly neutralized by the natural alkalinity of the water. (This is assumed to be calcium bicarbonate.)

$$3Ca(HCO_3)_2 + 3H_2SO_4 \rightarrow 3CaSO_4 + 6CO_2 + 6H_2O$$

(The acid may also be neutralized by the addition of an alkaline agent, commonly lime.)
The overall equation, assuming 15 molecules of water of crystallization, is often written as follows:

$$Al_2(SO_4)_3\ 15\ H_2O + 3Ca(HCO_3)_2 \rightarrow 2Al(OH)_3 + 3CaSO_4 + 6CO_2 + 15H_2O$$

Aluminum hydroxide (more accurately the hydrous oxide of aluminum) is an amphoteric substance. This means that under acid conditions (low pH) it behaves as a base and under alkaline conditions it behaves as an acid. Consequently it will be dissolved by too great a surplus of either H^+ or OH^- ions.

$$Al(OH)_3 + 3H^+ \rightarrow Al^{+++} + 3H_2O$$

(Solution at low pH)

$$Al(OH)_3 + OH^- \rightarrow AlO_2^- + 2H_2O$$

(Solution at high pH)

Precipitates formed by iron coagulants do not exhibit amphoteric behavior, at least not the extent shown by those derived from alum.

These reactions are intended to serve as simplified illustrations and do not purport to indicate the exact chemistry of the coagulation processes nor the composition of the alum floc.(5) The chemistry involved is much more complex than these simple equations suggest. However, certain observations of a qualitative nature may be made based on a consideration of these equations. There are:

a) Alum yields both plus and minus ions of fairly high charge.
b) Alum produces a precipitate. (The composition varies).(5)
c) Alum is an acid salt. Its hydrolysis can result in a reduction in the natural alkalinity of the water and an increase in the carbon dioxide content. The result is an increase in the hydrogen ion (H^+) concentration or a reduction in the pH value of the water.

d) Because of its amphoteric nature the solubility of alum floc is sensitive to the pH value of the water. pH adjustments may be required to achieve optimum conditions for precipitation.

Similar equations leading to generally similar conclusions could be written for most of the iron coagulants (ferric sulfate, ferric chloride, chlorinated copperas) Ferrous sulfate (copperas) behaves similarly; however, its reaction is further complicated by the necessity of oxidizing the iron from the "ferrous" to the less soluble "ferric" form. As previously noted an important difference between alum and the iron coagulants is the fact that the precipitates or flocs formed by the latter are not amphoteric.

Colloidal Particles

The coagulation process cannot be described in terms of simple, stoichiometric relationships. Coagulation is intimately associated with the science of colloidal materials. To a great extent understanding of the mechanism of coagulation is dependent upon knowledge of the characteristics and behavior of matter in the colloidal state. Colloid chemistry has been characterized as "the twilight between chemistry and physics."(6)

Colloidal dispersions may be thought of as consisting of a homogeneous medium such as water and particles dispersed therein. The term "colloid" does not refer to a particular kind of matter but to a somewhat indefinite state of subdivision. In general colloidal particles are not microscopically resolvable nor can they be readily separated from water by filtration methods. The author is not aware of any satisfactory, short definition of colloidal systems. No true dividing line exists between what may be termed suspensions, colloids, and solutions. Colloidal dispersions have been referred to as "colloidal suspensions" and as "colloidal solutions." Actually there is a "continuous transition" from suspensions through colloids and on to so-called true solutions.(6)

Particle Size

Colloidal particles are often arbitrarily considered to be those falling in the approximate diameter range 1.0 to 100 millimicrons (10^{-7} to 10^{-5}cm) although some authorities suggest a range of "over 1.0 millimicron to something less than 500 millimicrons."(6) Such distinctions are purely arbitrary and are based on expediency.

For comparative purposes the following table of approximate particle sizes is presented.

TABLE II

PARTICLE DIAMETERS (7)

Material	Diameter (Order of Magnitude)
Gravel	1 cm upward
Coarse Sand	0.1 cm (10^{-1}cm.)
Fine Sand	.01 cm (10^{-2}cm.)
Silt	10^{-3}cm.
Bacteria	10^{-4}cm. (1 micron)
Clay	10^{-5}cm.
Virus	10^{-7}cm. (1 millimicron)
Oxygen molecule	10^{-8}cm.

Surface Area

Some of the distinctive properties of colloidal particles can be explained by their large specific surface area. Assume that water contains 200 parts per million of a material having a specific gravity of 2.5 and that this material is colloidally dispersed in the water as spherical particles having a diameter of 1.0 millimicrons (10^{-6}cm). Each 1000 gallons of water would contain about 6 x 10^{20} particles and their surface area would be approximately 45 acres. Obviously surface activity plays a prominent role in colloidal behavior.

Charge

Colloidal particles normally bear a charge. This charge probably can be obtained in various ways, including ionization of a part of the colloidal particle. The charge may also be obtained as a result of the colloid's surface activity; i.e., by preferential adsorption of various ions from the water including hydrogen (H^+) or hydroxyl (OH^-) ions.(8) The stability of many colloids can be explained by the fact that the particles carry like charges. Similarly charged particles repel each other; thus, continued dispersion rather than precipitation is favored. The degree of stability of a particle is determined by a value termed the "zeta potential" which is related to particle charge and the distance through which the charge is effective.(8)

Particle charge can be demonstrated experimentally using colloidal material found in natural waters. For example Saville,(9) working with a colored water, showed that the color was caused by negatively charged colloidal particles. Saville actually demonstrated electrophoresis, a characteristic property of colloidal particles having to do with their migration under the influence of an electrical potential. Colloidal particles encountered in natural waters such as clay and organic color are commonly negatively charged. On the other hand colloids formed in the initial stages of the coagulation by aluminum and iron salts generally carry a positive charge.

Precipitation of Colloidal Particles

Colloidal particles can be precipitated by ions of opposite charge. The relative precipitating power of ions increases tremendously with increase in the ion's charge or valence. For example sulfate (SO_4^{--}) is more than 100 times as effective as chloride (Cl^-) in precipitating positively charged colloidal particles. The aluminum ion (Al^{+++}) is roughly 500 times as effective as the sodium ion (Na^+) for precipitating a negatively charged colloidal particle. The greater the concentration of dissolved mineral solids (ionic strength) the more readily precipitation occurs. (An increase in ion concentration lowers the zeta potential). Therefore it is generally easier to accomplish coagulation in waters of high mineral content than in those containing low concentrations of mineral matter.

Inasmuch as ions responsible for charge on colloidal particles are often hydroxyl (OH^-) or hydrogen ions (H^+) colloidal-particle precipitation should be influenced by the pH value of the water. Water treatment experience supports this statement. The process of coagulation is often quite sensitive to pH changes. Recognition of the importance of the pH factor in coagulation may mean the difference between good and poor results.

Coagulation Process

Mechanism of Coagulation

Black[3] whose studies of coagulation phenomenon have contributed greatly to our knowledge of the subject has divided the process into three phases.

a) The coagulant furnishes trivalent aluminum or ferric ions (high plus charge) capable of neutralizing the negative charge on colloidal particles of color and turbidity.

b) If the coagulation reaction occurs below about pH 7.0 the result of the initial action of the coagulant is the formation of positively charged particles of colloidal dimensions. Black has termed these "micro-flocs." Their plus charge at low pH is probably due to adsorption of hydrogen ions and surplus aluminum or ferric ions. These positively charged particles have adsorptive capacity and neutralizing power and can continue to remove negatively charged particles of turbidity and color.

c) The third or flocculation phase involves further adsorption, charge neutralization and finally agglomeration of the micro-flocs into larger and larger particles which will settle readily.

It is apparent that two types of colloids are involved in coagulation, namely, those present in the water undergoing treatment and those formed by the coagulating chemicals added to the water. In general the former are negatively charged whereas the latter commonly carry a plus charge. However reversal of the charge on particles formed by the initial coagulation reactions may occur at approximately pH 7.0 and above. This is probably due to adsorption of hydroxyl ions.

Factors Affecting the Process

pH Value

The pH value of the water affects the solubility of precipitates formed by iron and aluminum, time required for floc formation, and charge on colloidal particles. Effect of pH value on the time required for floc formation has been studied by Theriault and Clark;[10] Bartow, Black and Sansbury;[11] Black, Rice and Bartow;[12] Baylis[13] and numerous other investigators. Based on theoretical considerations and certain experimental evidence Theriault and Clark concluded that when other influences are not considered optimum conditions for alum-floc formation will be found at about pH 5.5. From a practical standpoint other influences are important but it is often the case that alum coagulation is best accomplished at fairly low pH values (5.0 - 7.0). This is particularly true of soft, colored waters.

The pH range of relative insolubility for aluminum is approximately 5.0 to 7.0, above pH 4.0 for ferric iron and above pH 9.5 for ferrous iron.[8]

Dissolved Mineral Matter

Dissolved mineral matter in the water is a source of ions which can play an important part in the coagulation process. Coagulation of the positive colloidal particles (micro-flocs) formed by iron or aluminum coagulants is influenced considerably by other ions. The importance of any particular ion is to a great extent related to its valence or charge. As previously mentioned, the sulfate ion which bears a fairly-high negative charge strongly favors precipitation of positively charged micro-flocs—much more so than the chloride ion which carries a lower charge. Various investigators have

demonstrated that sulfate definitely extends the zone of optimum floc formation in the lower pH range. The magnitude of this effect is somewhat dependent on sulfate concentration. Sulfate is a constituent of two commonly used coagulants, alum and ferric sulfate, and is found in a wide range of concentrations in natural waters.

At pH values of approximately 7.0 and above the charge on the micro-flocs may be negative rather than positive. If such is the case then positively charged ions particularly calcium and magnesium (high plus charge) would play a part in floc formation.

In the complex process of coagulation probably all ions have some influence. It is therefore apparent that natural waters, which vary widely in chemical make-up, will respond in various ways to the addition of coagulating chemicals.

Effects of synthetic detergents on coagulation have been investigated by Vaughn(25) and associates at the South District Filtration Plant, Chicago, and by Smith(26) and others at the Robert A. Taft Sanitary Engineering Center, Cincinnati.

Temperature

The principal effect of temperature is its influence on the rate of floc formation. Generally the lower the temperature the longer the time required to produce good flocculation. Higher coagulant dosages may be required for cold water than for warm. Powell(14) has presented curves illustrating increased chemical requirements at temperatures below 50° F. Camp's studies(15) showed that decreasing the temperature shifted the optimum pH to slightly higher values. Thuma(16) has reported recently on the adverse effects of low water temperature on water softening.

Time-Concentration

Various authorities(8)(17) have called attention to the fact that the coagulation process is a time-concentration phenomenon. High concentrations of coagulant produce results in shorter time than low concentrations.

Coagulant Aids

Discovery that partially neutralized sodium silicate solutions would substantially improve coagulation was a notable advance in water treatment.(13) The silica activation process yields a negatively charged colloidal particle which not only hastens the rate of coagulation but also results in the formation of larger floc particles. Activated silica is also beneficial in connection with water softening by the lime-soda ash method and has been successfully used as a substitute for the conventional coagulants.(18)

Lime, soda ash and acids (sulfuric and phosphoric) are coagulant aids in the sense that they can be used when necessary for pH adjustment.

Suspended particles can assist flocculation by acting as nuclei for floc formation. Coagulation in some instances may be improved by the addition of small amounts of finely divided, insoluble material. Clay, bentonite, and activated carbon have been used for this purpose.(17) Nuclei may also be provided by returning sludge from previously treated water.

Application of chlorine ahead of the coagulation process has been reported to be advantageous.(14)(19) Chlorine may alter or destroy materials which inhibit floc formation and render other materials more susceptible to coagulation.

Polyelectrolytes are a recent development of considerable interest. These

are synthetic organic compounds and are said to be useful in flocculating a wide variety of suspensions. One application has been in the treatment of suspensions encountered in ore processing. Apparently they can be used alone or in combination with conventional coagulants.

Exchange Capacity of Turbidity Particles

Investigations by Langelier and associates(20)(21) have indicated that an important factor in the coagulation process is the cation exchange capacity of the turbidity particles. When exchange capacity is fairly low, flocculation requires a binder material such as alum or iron floc. An amount of coagulant proportional to the exchange capacity must be added to provide colloid-destabilizing cations (Al^{+++} or Fe^{+++}) and an additional amount is required to furnish binder material. Formation of the latter by coagulant hydrolysis occurs only after the turbidity particles have been destabilized by the coagulant cations.

Mixing

The importance of appropriate mixing cannot be over-emphasized. Without it efficient coagulation is virtually impossible. It is important to recognize and distinguish between the two types of mixing commonly employed in coagulation. These are "rapid" or "flash" mixing and "slow" mixing or flocculation. Rapid mixing requires only a few seconds using a high-speed mixing device. Flocculation on the other hand requires a much longer period of mixing at fairly low velocity.

The first and probably one of the most important phases of coagulation involves the action of aluminum or ferric ions provided by the coagulant. It is therefore important that rapid dispersion of the coagulant through the water be achieved. The function of a rapid mixing device is that of quick and thorough dispersion of the coagulant.

The initial phase of coagulation produces finely divided particles and opportunity for coalescence must be provided if a readily settleable floc is to be obtained. Coalescence requires contact between particles and there is greatly increased opportunity for such contact if the water is slowly mixed. The result of slow-mixing is floc "build-up" or "flocculation." (The process is often referred to as "conditioning.")

Flocculating velocities employed vary but often range from about 0.9 to 1.5 feet per second.(22) Too high a velocity will prevent the formation of large floc particles or break up those that do form. Flexibility in mixing speed is desirable in most instances. Time allowed for slow mixing varies but an allowance of 30 to 45 minutes is common practice. Camp(23) has related the rate of flocculation to velocity gradient and detention time.

Following the flocculation phase of the coagulation process turbulence must be avoided. The floc particles are fragile and can be broken up in passage from the flocculation chamber to the settling basin. The transition from flocculation to sedimentation should be as direct as possible and be accomplished at low velocity. If conduits are required velocity in them should in general be less than 1.0 foot per second.

Coagulation Control

In view of the complexity of the coagulation process and the well-known variability of natural waters, it is not surprising that coagulation poses a

difficult control problem for the water-plant operator. For practical operating control there is no substitute for trial determinations of coagulant dosage conducted with laboratory stirring equipment.(24) Optimum dosage, pH effects, and use of coagulation aids can be readily studied in the laboratory and it is essential that this be done. The results of laboratory studies, to be useful, have to be interpreted in terms of actual plant performance. For this, there is no substitute for experience. In this connection one further point needs emphasis. Laboratory data indicating the need for adjustments are of little value to an operator if the means of making the required changes are not available to him. Obviously there is often need for the provision of flexibility in the way of chemicals and facilities for their application, points of chemical feed, and mixing. The coagulation process cannot be forced into a rigid pattern. Although coagulating chemicals have been used in large-scale water treatment for more than a half century, we still have much to learn about their behavior and the nature of the substances dispersed in natural waters.

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A FLOW CONTROLLER FOR OPEN OR CLOSED CONDUITS

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(Proc. Paper 1037)

SYNOPSIS

A general principle of flow control, utilizing a non-linear resistance, is eveloped that is applicable to both open and closed conduits. The control is nfinitely adjustable for discharge over its design range, and holds the flow lose to the predetermined discharge over its head range. The control is eadily converted into a flow meter with a linear head-discharge curve and djustable range and sensitivity. Experimental work is presented to confirm ne theory.

INTRODUCTION

The combination of a disc moving within a profiled throat, and a nonlinear esistance to support the disc against the pressure drop is shown to yield a rinciple of flow control that is infinitely variable and theoretically exact (i.e., ith no change in discharge or drag coefficients). By reversing the throat ection and measuring the head across the disc, the device is converted into a ow meter. The underlying principles of single orifice flow control are out- ned briefly before developing the specific relationships and design data. A nodel has been constructed and tested in the Civil Engineering hydraulic aboratory of the University of Michigan. The results are presented for both ne flow control and the flow meter.

Single Orifice Flow Control Concepts

An orifice flow control must act to hold the discharge, Q, constant in

$$Q = C_q \ A\sqrt{2g\ h} \tag{1}$$

y changing the orifice area, A, as the head, h, varies over its design range.

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C_q is the discharge coefficient, and g the acceleration due to gravity. Assu ing the discharge coefficient constant, the area must vary inversely as the square root of the head. For a control meant to operate at a single dischar only, this may be accomplished by means of a linear spring assembly, Fig. Head across the device displaces the disc, thereby altering the net area. T throat may be profiled so that the discharge is held constant.

If one attempts to make a variable flow control from this device, by vary the position, z, of the spring base BB, Fig. 1, the dimensionless head-dis- charge curves appear as in Fig. 2, with Q_d the design discharge, h_o the mi mum design head, and η the ratio of maximum design head to minimum des head.

By selecting another approach, although still retaining a linear spring, t throat profile may be constructed so that the discharge is the same at mini- mum and maximum heads, for any setting, z, Fig. 1, within its size range. The discharge at intermediate heads varies from that at the end points, how ever, as indicated in Fig. 3. To obtain the area required, as a function of x (Fig. 1), let Y be the displacement of the disc for head change from h_o to h_o i.e., from minimum to maximum design head. The area must change from A_1 to A_2 as given by

$$A_1\sqrt{h_o} = A_2\sqrt{h_o\eta} \qquad ($$

which must hold for any setting, z, of the spring base BB. As the area of or fice opening is a function of x only, $A = f(x')$,

$$f(x')\sqrt{h_o} = f(x' - Y)\sqrt{h_o\eta} \qquad ($$

Since this relation must hold for any initial x', a law of variation of A with x of the following form is indicated

$$A = A_o e^{\alpha x'} \qquad ($$

where A_o, α, are undetermined constants. Substituting into Eq. (3)

$$A_o e^{\alpha x'}\sqrt{h_o} = A_o e^{\alpha(x' - Y)}\sqrt{h_o\eta}$$

or, simplifying

$$\alpha = \frac{\ln\eta}{2Y} \qquad ($$

where ln is the natural logarithm.

The variation of A with x' then becomes

$$A = A_o e^{\frac{x'\ln\eta}{2Y}} \qquad ($$

with A_o the opening between disc and throat when x = 0, i.e., $A_o = A_{min}$.

To improve the accuracy of this control for the same head and discharge range, two linear springs may be introduced into the assembly, as in Fig. 4, altering the first spring so that the disc is displaced $\frac{Y}{2}$ by the first spring o as the head varies from h_o to $h_o\sqrt{\eta}$ and is displaced the remaining distance as both springs act with the head changing from $h_o\sqrt{\eta}$ to $h_o\eta$. This results a dimensionless head-discharge curve, Fig. 5, in which the control is accur for three heads and the maximum error has been reduced.

By taking n linear springs, the first acting over the distance Y/n, the fir three over 2Y/n to 3Y/n, etc. with the corresponding head ranges

h_o to $h_o\eta^{1/n}$, $h_o\eta^{1/n}$ to $h_o\eta^{2/n}$, $h_o\eta^{2/n}$ to $h_o\eta^{3/n}$, etc.

the discharge is maintained the same at n + 1 heads with the maximum variation in discharge decreasing as n increases. By increasing n indefinitely, a theoretically exact flow controller is obtained. The spring system then becomes a non-linear spring. The relationships for this case are developed in the following section.

Basic Equations for Nonlinear Resistance Flow Control

The basic equations for the flow controller have been derived by the author in a different manner in a previous paper.[1] One form of the nonlinear resistance flow control is shown in Fig. 6. The spring element is of the restrained-tip cantilever type, in which the two leaf springs are forced together by a profiled backing that shortens their effective length. The spring element is a distance y = Y apart at minimum design head and contacts at y = 0 for maximum design head. By rotating the milled knob, the discharge setting, z, can be varied. From the geometry of Fig. 6 it is evident that

$$x' = y + z \tag{7}$$

where x' gives the disc position (the area of opening between disc and throat is a function of x' only); and y is a function of the pressure drop across the disc only, i.e., it is a function of h only, for a given fluid.

For any setting z

$$A\sqrt{h} = (A + \Delta A)\sqrt{h + \Delta h} \tag{8}$$

or, since A = f(x')

$$\sqrt{h}\, f(x') = \sqrt{h + \Delta h}\, f(x' + \Delta y) \tag{9}$$

because a change in Δh causes a change in Δy and $\Delta x' = \Delta y$ for constant z. Expressing the area of opening, A, as an exponential function of x', Eq. (4)

$$f(x') = A = A_{min}\, e^{\alpha x'} \tag{10}$$

where α is as yet undetermined. Substituting Eq. (10) into Eq. (9), and solving for α,

$$-\alpha = \frac{\ln\sqrt{1 + \frac{\Delta h}{h}}}{\Delta y}$$

Expanding the logarithmic term into a series and taking the limit as Δy approaches zero,

$$-\alpha = \lim_{\Delta y \to 0} \frac{1}{2}\left[\frac{1}{h}\frac{\Delta h}{\Delta y} - \frac{1}{2h^2}\frac{\Delta h^2}{\Delta y} + \cdots\cdots\right]$$

or

$$-\alpha = \frac{1}{2h}\frac{dh}{dy} \tag{11}$$

Since α is a function of x' only, and y is a function of h only, Eq. (11) shows a function of x' equal to a function of y'. This is impossible because of Eq. (7),

1. "A Quick Response Variable Flow Control Device," V. L. Streeter, Instrument Society of America, Vol. 2, No. 2, Feb. 1955, pp. 48-51.

hence both sides of Eq. (11) must be constant. Integrating Eq. (11)

$$-2\alpha y = \ln h + c$$

Using the end conditions: $h = h_o$, $y = Y$; $h = h_o\eta$, $y = o$, both α and c are determined,

$$\alpha = \frac{\ln \eta}{2Y} \tag{12}$$

and

$$y = \frac{Y}{\ln \eta} \ln \frac{h_o \eta}{h} \tag{13}$$

Equation (10) becomes

$$A = A_{min}\, e^{\frac{x' \ln \eta}{2Y}} \tag{14}$$

Now, substituting into Eq. (1), using h from Eq. (13), A from Eq. (14) and x' from Eq. (7)

$$Q = C_q\, A_{min} \sqrt{2g\, h_o \eta}\; e^{\frac{z \ln \eta}{2Y}} \tag{15}$$

which yields the discharge for the given arbitrary setting z, and shows that Q is independent of head h.

The three equations, Eqs. (13), (14), and (15) determine the characteristic of the flow control. The nonlinear resistance must follow the head-deflection relation of Eq. (13), and the throat profile must be made so that the net area of opening is given by Eq. (14). By providing for the base of the resistance to be adjusted axially, the quantity z may be changed and variable flow control results, as given by Eq. (15).

Equation for Throat Profile

If an axially symmetric throat section is desired, the throat profile, as given by the cylindrical coordinates (r, x) in Fig. 7, must yield the net area for any x' required by Eq. (14). Using Pappus theorem this area is

$$A = 2\pi \frac{r_o + r}{2} \sqrt{(r - r_o)^2 + (x' - x)^2}$$

$$= A_{min}\, e^{\frac{x' \ln \eta}{2Y}} \tag{16}$$

or

$$(r'^2 - r_o^2)\, e^{\frac{x' \ln \eta}{2Y}} = (r_o + r) \sqrt{(r - r_o)^2 + (x' - x)^2} \tag{17}$$

For a given x' this equation yields a curve of r as a function of x. Drawing segments of these curves for various values of x', Fig. 8, shows that their <u>envelope</u> yields the desired throat profile. The partial differential of Eq. (17) with respect to x' is taken and solved simultaneously with Eq. (17) to yield the cylindrical coordinates of the throat profile. Dividing the partial of Eq. (17) with respect to x' into Eq. (17) yields,

$$x' - x = \frac{Y}{\ln \eta}\left(1 - \sqrt{1 - \left[\frac{(r - r_o)\ln \eta}{Y}\right]^2}\right) \tag{18}$$

Substituting Eq. (18) into Eq. (17) gives the desired equation,

$$x = \frac{2Y}{\ln\eta}\left\{\ln\left[\frac{r_o + r}{r'^2 - r_o^2}\sqrt{2}\,\frac{Y}{\ln\eta}\sqrt{1-\sqrt{1-\left[\frac{(r-r_o)\ln\eta}{Y}\right]^2}}\right] - \frac{1}{2}\left[1-\sqrt{1-\left[\frac{(r-r_o)\ln\eta}{Y}\right]^2}\right]\right\} \tag{19}$$

This equation permits the direct computation of x for any r in terms of the design quantities Y, η, r', and r_o.

There is an inherent limitation to the value of r in Eq. (19), obtained by setting the radical term equal to zero,

$$r_{max} = r_o + \frac{Y}{\ln\eta} \tag{20}$$

For larger values of r the envelope solution breaks down. For $r = r_{max}$, from Eq. (19)

$$x_{max} = \frac{2Y}{\ln\eta}\left[\ln\left(\frac{2r_o + \frac{Y}{\ln\eta}}{r'^2 - r_o^2}\sqrt{2}\,\frac{Y}{\ln\eta}\right) - \frac{1}{2}\right] \tag{21}$$

The corresponding value of x', from Eqs. (18) and (21), is

$$x'_{max} = \frac{2Y}{\ln\eta}\ln\left(\frac{2r_o + \frac{Y}{\ln\eta}}{r'^2 - r_o^2}\sqrt{2}\,\frac{Y}{\ln\eta}\right) \tag{22}$$

Discharge Limitations

For the flow control to operate the disc must stay within the range $o \leq x' \leq x'_{max}$. For control over the whole head range this restricts z to the range $o \leq z \leq x'_{max} - Y$. To find the maximum discharge over the whole head range, $z = x'_{max} - Y$ is substituted into Eq. (15) using Eq. (22),

$$Q_{max} = C_q\, 2\pi\sqrt{g\, h_o}\,\frac{Y}{\ln\eta}\left(2r_o + \frac{Y}{\ln\eta}\right) \tag{23}$$

making use of the fact that $A_{min} = \pi(r'^2 - r_o^2)$. The minimum discharge is obtained from Eq. (15) by setting $z = o$,

$$Q_{min} = C_q\, A_{min}\sqrt{2g\, h_o\,\eta} \tag{24}$$

Equations (23) and (24) permit the selection of the three design quantities r_o, r', and Y for given head and discharge range. It is of interest to note that the maximum and minimum discharges are not related. The minimum throat radius r' does not come into Eq. (23).

For flow control over partial head ranges, the limitation is that z varies from -Y to o in the lower extended zone and from $x'_{max} - Y$ to x'_{max} in the upper extended zone, with the disc (x') remaining between o and x'_{max} at all times. The spring travel is limited to portions of the range Y to o, which restricts the head range to part of h_o to $h_o\eta$.

For z less than zero, the "threshold" value of y, to maintain $x' = o$, is

$$y = -z$$

Substituting this value of z into Eq. (15), using Eq. (13) for y,

$$Q = C_q \, A_{min} \sqrt{2gh} \tag{25}$$

This is the lower limit line of the lower extended zone.

For z greater than $x'_{max} - Y$, and with the disc at its limiting position $x' = x'_{max}$,

$$z = x'_{max} - y$$

Again, using Eqs. (15) and (13), the upper extended zone is limited by

$$Q = C_q \, A_{min} \sqrt{2gh} \; e^{\frac{x'_{max} \ln \eta}{2Y}} \tag{26}$$

The total flow control range is shown in Fig. 9 in dimensionless form, with the z range indicated on the right side.

Design of Nonlinear Resistance

The nonlinear resistance to displacement of the disc, as required by Eq. (13), may be expressed in terms of force exerted on the disc. This force, F, is

$$F = C_d \pi r_o^2 wh \tag{27}$$

where C_d is the dimensionless drag coefficient, and w is the unit weight of liquid. The minimum design force, F_o, is then

$$F_o = C_d \pi r_o^2 w h_o \tag{28}$$

C_d is assumed to be constant over the range of travel of the disc. Substituting for h and h_o in Eq. (13)

$$y = \frac{Y}{\ln \eta} \ln \frac{F_o \eta}{F} \tag{29}$$

The resistance may be obtained in many ways, three of which are discussed here, namely 1) fluid resistance, 2) dead weight resistance, and 3) spring resistance.

<u>Fluid Resistance</u>. Referring to Fig. 10, a bellows with area, A_b forces manometer liquid into a manometer leg of varying cross-section. The area, a, of manometer is determined as a function of s. The necessary equations are Eq. (29),

$$-A_b \, dy = a_1 ds_1 = a \, ds \tag{30}$$

and

$$F = A_b \, (w_o - w) \, (s + s_1) \tag{31}$$

where a_1 is the constant cross-sectional area and w_o is the unit weight of manometer fluid. F and y may be eliminated from the last three equations to yield a and s in terms of s_1

$$a = \frac{a_1}{\frac{a_1 \ln \eta}{A_b Y} s_o \, e^{\frac{s_1 a_1 \ln \eta}{A_b Y}} - 1} \tag{32}$$

and

$$s = s_o e^{\frac{s_1 a_1 \ln\eta}{A_b Y}} - s_1 \tag{33}$$

where s_o is the value of s for $s_1 = o$ and $F = F_o$. The discharge setting is altered by changing z. For vertical installations the weight of moving parts would change the setting z for a given discharge, but would not otherwise disturb proper functioning of the controller.

Dead Weight Resistance. A weight suspended on a cable or metal tape such that it is lifted out by a cam can easily be arranged to yield the nonlinear force-displacement relation required by Eq. (29). Figure 11 illustrates one method, with the cam rotating about a transverse axis. To determine the shape of the cam, reference is made to Fig. 12, where the cam rotates through the angle, α, from o when $F = F_o$, to α_o when $F = F_o\eta$. The moment of the weight, W, about the shaft for any angle α is

$$W\rho\cos(\theta-\alpha) = F\rho_o = F_o\rho_o\eta\, e^{-y\frac{\ln\eta}{Y}} \tag{34}$$

With ρ_o constant,

$$\frac{\alpha}{\alpha_o} = 1 - \frac{y}{Y}$$

since $\rho_o\alpha = Y - y$. Letting $W = F_o$, Eq. (34) reduces to

$$\rho\cos(\theta-\alpha) = \rho_o e^{\frac{\alpha\ln\eta}{\alpha_o}} \tag{35}$$

For any α, Eq. (35) is a straight line in polar coordinates (ρ, θ). The envelope of the family of straight lines is the cam profile equation. Taking the partial differential of Eq. (35) with respect to α and solving simultaneously with Eq. (35) to eliminate α yields

$$\rho = \rho_o\sqrt{1+\left(\frac{\ln\eta}{\alpha_o}\right)^2}\; e^{-\frac{\ln\eta}{\alpha_o}\tan^{-1}\frac{\ln\eta}{\alpha_o}}\; e^{\frac{\theta\ln\eta}{\alpha_o}} \tag{36}$$

which is a logarithmic spiral. By dividing the partial differential of Eq. (35) with respect to α by Eq. (35)

$$\tan(\theta-\alpha) = \frac{\ln\eta}{\alpha_o} \tag{37}$$

showing that $\theta - \alpha$ is a constant. θ varies from $\tan^{-1}(\ln\eta/\alpha_o)$ to $\alpha_o + \tan^{-1}(\ln\eta/\alpha_o)$ in Eq. (36).

If a cam is to be designed to support the submerged weight, G, of the moving parts of the controller,

$$F_o\rho_o e^{\frac{\alpha\ln\eta}{\alpha_o}} + G\rho_o = W\rho\cos(\theta-\alpha) \tag{38}$$

Taking the partial differential with respect to α and dividing by Eq. (38)

$$\alpha_o\tan(\theta-\alpha) = \frac{\ln\eta}{1+\frac{G}{F_o}e^{-\frac{\alpha}{\alpha_o}\ln\eta}} \tag{39}$$

For a series of values of α between o and α_0, corresponding values of θ are found. Using these values the corresponding values of ρ are found in Eq. (38), yielding polar coordinates of the cam profile.

Spring Resistance. A restrained-tip cantilever spring may be designed to provide the required resistance law by following a method due to Mr. S. P Clurman.[2] Two identical, flat springs of constant width and thickness are placed as shown in Fig. 13. As the force, P, is increased the springs are forced against the profiled backings, thereby reducing their effective lengths and increasing their stiffness. The springs are fastened together at each end by a spacer block that prevents rotation. As there are actually four springs, each one takes half the load and is deflected through half the total displacement. The equation of the backing profile is determined as follows:

Letting $G = kF_o$ be the submerged weight of moving parts of the controller for vertical stem applications, (G = 0 for horizontal applications)

$$P = F - G = F_o \left(\eta e^{\frac{-y \ln \eta}{Y}} - k\right)$$

The sign of G has been taken for the case where the weight of moving parts acts in the opposite direction to the pressure force on the disc. With δ the deflection of one spring from the position for minimum design head

$$\delta = \frac{1}{2} (Y - y) = \frac{Y}{2 \ln \eta} \ln \left(\frac{2P'}{F_o} + k\right) \tag{40}$$

where P' is the load on a single spring (P/2). For the minimum design load the spring is cantilevered from its full length, ℓ, with the backing touching at $x_o = o$, Fig. 14. The variables x_o, y_o are coordinates of the backing profile. With no load on the spring it will be displaced from the $\delta = 0$ position a distance

$$-\frac{F_o(1-k)\ell^3}{24\ EI}$$

where E is the modulus of elasticity and I the section moment of inertia about the neutral axis.

With (x_o, y_o) the last point of contact between spring and backing for a given active load P', the deflection from no-load position is

$$\delta + \frac{F_o(1-k)\ell^3}{24\ EI} = \frac{(\ell-x_o)^2}{6} \frac{d^2y_o}{dx_o^2} + \frac{2}{3} (\ell-x_o) \frac{dy_o}{dx_o} + y_o \tag{41}$$

The right-hand side of the equation is obtained by consideration of deflection of cantilever beams, as follows: In Fig. 15, by superposition of the two simple loadings

$$\frac{dy_o}{dx_o} = \frac{P'(\ell-x_o)^2}{2\ EI} - \frac{M(\ell-x_o)}{EI} \tag{42}$$

$$\text{Defl.} = \frac{P'(\ell-x_o)^3}{3\ EI} - \frac{M(\ell-x_o)^2}{2\ EI}$$

Replacing M by EI d^2y_o/dx_o^2 and eliminating P' in the two equations

2. "The Design of Nonlinear Leaf Springs", by S. P. Clurman, Transactions, American Society of Mechanical Engineering, Feb. 1951, pp. 155-161.

$$\text{Defl.} = \frac{1}{6}\,(\ell - x_o)^2\,\frac{d^2y_o}{dx_o{}^2} + \frac{2}{3}\,(\ell - x_o)\frac{dy_o}{dx_o}$$

which, with the addition of y_o, is the right-hand side of Eq. (41).

δ and F_o may be expressed in terms of the stiffness of the spring. From Eq. (40)

$$p' = \frac{F_o}{2}\,\left(e^{\frac{2\delta \ln\eta}{Y}} - k\right)$$

Then

$$\frac{dP'}{d\delta} = \frac{F_o \ln\eta}{Y}\, e^{\frac{2\delta\ln\eta}{Y}} \tag{43}$$

which is the spring stiffness. For $\delta = 0$

$$\left.\frac{dP'}{d\delta}\right|_{\delta = 0} = \frac{F_o \ln\eta}{Y} = K_o = \frac{12\ EI}{\ell^3} \tag{44}$$

The stiffness of a spring varies inversely as the cube of its length, hence

$$\frac{dP'}{d\delta} = \frac{\ell^3 K_o}{(\ell - x_o)^3}$$

Solving Eq. (43) for δ, using the last two equations,

$$\delta = \frac{Y}{2\ \ln\eta}\ \ln\left[\frac{Y}{F_o \ln\eta}\cdot\frac{dP'}{d\delta}\right] = \frac{3Y}{2\ \ln\eta}\ \ln\frac{\ell}{\ell - x_o} \tag{45}$$

From Eq. (44)

$$\frac{F_o(1-k)}{24\ EI} = (1-k)\ \frac{Y}{2\ \ln\eta}$$

Substituting the last two equations in Eq. (41)

$$\frac{3Y}{2\ \ln\eta}\ \ln\frac{\ell}{\ell - x_o} + (1-k)\ \frac{Y}{2\ \ln\eta} = \frac{1}{6}(\ell - x_o)^2\frac{d^2y_o}{dx_o{}^2} + \frac{2}{3}(\ell - x_o)\frac{dy_o}{dx_o} + y_o \tag{46}$$

Inserting the dimensionless quantities

$$\bar{x} = \frac{x_o}{\ell}.\quad \bar{y} = \frac{y_o}{Y}$$

$$\frac{3(1-k)}{\ln\eta} - \frac{9}{\ln\eta}\ \ln(1-\bar{x}) = (1-\bar{x})^2\ \frac{d^2\bar{y}}{d\bar{x}^2} + 4(1-\bar{x})\ \frac{d\bar{y}}{d\bar{x}} + 6\bar{y}$$

This equation is converted into a linear equation with constant coefficients by the substitution of

$$\zeta = \ln(1-\bar{x})$$

Then

$$\frac{d\bar{y}}{d\bar{x}} = -e^{-\zeta}\ \frac{d\bar{y}}{d\zeta}\ ,\quad \frac{d^2\bar{y}}{d\bar{x}^2} = e^{-2\zeta}\left(\frac{d^2\bar{y}}{d\zeta^2} - \frac{d\bar{y}}{d\zeta}\right)$$

and Eq. (47) becomes

$$\frac{d^2\bar{y}}{d\zeta^2} - 5\frac{d\bar{y}}{d\zeta} + 6\bar{y} = \frac{3}{\ln\eta}(1-k-3\zeta) \tag{48}$$

The solution of this equation for the conditions

$$\bar{y} = 0,\ \zeta = 0,\ \frac{d\bar{y}}{d\zeta} = 0$$

is

$$2\bar{y}\ln\eta = -k - \frac{3}{2} - 3\zeta + 3\left(\frac{1}{2} + k\right)e^{2\zeta} - 2ke^{3\zeta}$$

Substituting back for $\bar{x}$,

$$2\bar{y}\ln\eta = -k - \frac{3}{2} - 3\ln(1-\bar{x}) + 3\left(\frac{1}{2}+k\right)(1-\bar{x})^2 - 2k(1-\bar{x})^3 \tag{49}$$

This is the dimensionless equation for spring backing profile.

From Eq. (45) the maximum value of $\bar{x} = x_o/\ell$ is, for $\delta = Y/2$,

$$\bar{x}_{max} = 1 - \eta^{-1/3} \tag{50}$$

The cross-section of the spring must satisfy Eq. (44).

Consideration of stresses in the spring indicates the maximum moment at the spring extremity. From Fig. 16 the moment M_1 is given by

$$M_1 = P'\ell(1-\bar{x}) - M_2 \tag{51}$$

M_2 may be found from Eq. (42), by obtaining dy_o/dx_o from Eq. (49)

$$\frac{dy_o}{dx_o} = \frac{Y}{\ell}\frac{d\bar{y}}{d\bar{x}} = \frac{Y}{2\ell\ln\eta}\left[\frac{3}{1-\bar{x}} - 3(1+2k)(1-\bar{x}) + 6k(1-\bar{x})^2\right]$$

and

$$\frac{M_2}{EI}\ell(1-\bar{x}) = \frac{P'\ell^2}{2\ EI}(1-\bar{x})^2 - \frac{Y}{2\ell\ln\eta}\left[\frac{3}{1-\bar{x}} - 3(1+2k)(1-\bar{x}) + 6k(1-\bar{x})^2\right] \tag{52}$$

From Eqs. (40) and (45)

$$P' = \frac{F_o}{2}\left(\frac{1}{(1-\bar{x})^3} - k\right)$$

and from Eq. (44)

$$\frac{Y}{\ln\eta} = \frac{F_o\ell^3}{12\ EI}$$

Substituting the last two equations into Eq. (52) and simplifying

$$M_2 = \frac{F_o\ell}{8}\left[\frac{1}{(1-\bar{x})^2} + 1 - 2k(1-\bar{x})\right]$$

Replacing M_2 in Eq. (51) by this value,

$$M_1 = \frac{F_o\ell}{8}\left[\frac{3}{(1-\bar{x})^2} - 1 - 2k(1-\bar{x})\right]$$

M_1 is a maximum for $\bar{x} = 1 - \eta^{-1/3}$,

$$M_{1_{max}} = \frac{F_o \ell}{8} (3\eta^{2/3} - 1 - 2k\eta^{-1/3}) \quad (53)$$

The maximum fibre stress, S_m, is

$$S_m = \frac{t\ M_{1max}}{2\ I} = \frac{3}{4} \frac{F_o \ell}{b\ t^2} (3\eta^{2/3} - 1 - 2k\eta^{-1/3}) \quad (54)$$

where t is the spring thickness and b the spring width. Eqs. (44) and (54) permit the thickness and width to be expressed in terms of S_m, ℓ, and the design quantities η, Y, k, and F_o,

$$\frac{t}{\ell^2} = \frac{4}{3} \frac{S_m}{EY} \frac{\ln\eta}{(3\eta^{2/3} - 1 - 2k\eta^{-1/3})} \quad (55)$$

and

$$b\ell^3 = \frac{27}{64} \frac{F_o(EY)^2}{S_m^3 (\ln\eta)^2} (3\eta^{2/3} - 1 - 2k\,\eta^{-1/3})^3 \quad (56)$$

The restrained-tip cantilever spring is relatively simple to construct and has very small hysteresis loss. Its main objection, from the standpoint of a closed conduit controller, is its size.

Flow Meter with Adjustable Sensitivity and Range

If the throat casting in the flow control unit is reversed, as in Fig. 17, and a manometer is connected to the two ends of the throat, the pressure drop across the disc, as given by R the gage difference, is a linear function of the discharge. The relation among the variables x', y, z' is

$$x' + y = z' \quad (57)$$

where z' + z is a constant. Substituting into Eq. (1), using Eqs. (13), (14) and (57) to eliminate x', y, and A,

$$Q = C_q\ A_{min} \sqrt{\frac{2g}{h_o\eta}}\ e^{\frac{z' \ln\eta}{2Y}}\ h \quad (58)$$

showing Q to be a linear function of h.

The sensitivity and range is changed by adjusting z, or z'. Equation (58) may be arranged into the dimensionless form

$$\frac{Q}{C_q A_{min}\sqrt{2g h_o \eta}} = \frac{e}{\eta}^{\frac{z' \ln\eta}{2Y}} \frac{h}{h_o} \quad (59)$$

which is plotted in Fig. 18.

The upper threshold of the metering range is given by

$$\frac{Q}{C_q A_{min}\sqrt{2g h_o \eta}} = \frac{e}{\eta}^{\frac{x'_{max} \ln\eta}{2Y}} \sqrt{\frac{h}{h_o}} \quad (60)$$

The lower threshold of the metering range is

$$\frac{Q}{C_q A_{min} \sqrt{2g\, h_{o\eta}}} = \sqrt{\frac{h}{h_{o\eta}}} \tag{61}$$

Large settings, z, result in large gage differences for a relatively small discharge range, while small settings permit metering over a much wider flow range. The sensitivity of the meter, as given by Q/h, is an exponential function of the setting z.

Experimental Program

An experimental flow control unit was designed to operate in a concrete flume in the floor of the Civil Engineering Hydraulic Laboratory. The unit was placed into a circular opening in the horizontal shelf of a barrier, Figs. 19 and 20. A 45° V-notch weir was placed a short distance downstream from the controller. Hook gages were mounted upstream and downstream from the barrier, so that the head across the controller, as well as the discharge, could be determined. The disc has a displacement of 0.75 inches as the head varies from a minimum of 2 inches to a maximum of 8 inches. The disc diameter is 7.50 inches and the minimum throat diameter was designed to be 7.60 inches. The minimum throat section was later measured to be 7.605 inches.

With the design quantities Y = 0.75 inches, h_o = 2 inches, η = 4, r_o = 3.75 inches, and r' = 3.80 inches, cylindrical coordinates of the throat profile were computed from Eq. (19). The restrained tip cantilever spring, Fig. 19, was used for the nonlinear resistance, and was computed from Eqs. (49), (55), and (56) for ℓ = 3 inches and k = 0, i.e. not allowing for weight of the disc assembly. The submerged weight of disc assembly was balanced by use of styrofoam floats placed on a downstream extension of the shaft. The amount of buoyancy was adjusted so that y = 0.75 inches when h = 2 inches. Originally the drag coefficient for the disc was assumed to be unity. Based on this coefficient the spring bottomed at h = 6.5 inches. By adding extra width to the spring the resistance was increased so that the disc displacement was 0.75 inches for head change from 2 inches to 8 inches.

The styrofoam floats also served another purpose by stabilizing the disc for the larger discharges. Without stabilization the disc tended to oscillate through its whole stroke for discharges greater than about 50 g.p.m. A steel can was anchored to the floor of the flume concentric with the disc and floats with about 0.25 inches radial clearance between can and floats. The resistance to forcing water into and out of the can provided adequate stabilization for all the experiments. Limitations in flume height restricted the range of heads for the larger discharges.

Fins were placed in the throat section to guide the disc. These were foun[d] to cause excessive friction, and were cut back radially to avoid contact with the disc. A teflon bearing in a spider at the small end of the throat provided excellent guidance with a minimum of resistance.

Test results for the flow controller are shown in dimensionless form in Fig. 21, with a summary of results in Table I.

The throat casting was inverted to convert the unit into a flow meter. He[ad] across the disc was measured with hook gages and discharge by means of th[e] weir. Test results are shown in dimensionless form in Fig. 22, with a summary of results in Table II.

TABLE I

SUMMARY OF RESULTS---FLOW CONTROLLER

Number of turns on adjusting screw	Head Range inches	Discharge g.p.m.	Maximum Divergence percent
0	2.32-8.27	20.0	1.4
3	2.17-8.03	24.5	1.6
6	2.22-8.12	30.0	2.0
9	2.12-8.72	35.8	2.2
12	2.34-8.06	44.8	1.3
15	2.20-8.88	54.5	2.8
18	2.29-8.46	66.6	1.9
21	2.14-7.91	80.8	2.4
24	1.92-7.59	99.4	2.8
27	1.88-7.17	123.8	3.9

TABLE II

SUMMARY OF RESULTS---FLOW METER

Number of turns on adjusting screw	$m = \frac{Q_{gpm}}{h_{inches}}$	Maximum Divergence percent
0	4.229	2.8
3	5.225	4.2
6	6.559	3.3
9	8.096	3.2
12	9.876	3.2
15	11.972	2.4
18	14.773	3.1
21	18.334	3.0
24	22.728	1.5
27	28.372	3.7

Discussion of Experimental Data

As only one model was constructed, there has been no opportunity to take advantage of experimental findings to increase accuracy. If a change in discharge coefficient occurs due to change of disc position within the throat, it may be compensated by redesigning the throat so that C_qA varies with x' as prescribed by the theory. In the present model no instrumentation was provided to determine the exact position of the disc for a particular setting. The setting z could be altered either by rotating the disc on its shaft (4 threads to the inch) or by rotating the adjusting knob over the spring (13 threads to the inch). The turns referred to in Figs. 21 and 22 are for the adjusting knob above the spring.

Flow Controller. For the flow controller the "0 turn" disc position at h = 8 inches was about at the minimum throat section. The average value of discharge for this setting, 20.05 g.p.m. was taken as Q_{min}. As the exact position of the disc in the throat is not known for this setting the area is unknown and likewise the discharge coefficient.

Separate measurements of discharge coefficient were made with the disc in known, fixed positions, and without the upper supporting framework for the spring assembly. These results indicated a coefficient of about 0.62 within the throat and about 0.63 with the disc slightly above the throat casting.

In each series of observations for flow controller or flow meter measurements were taken about equally for increasing heads and for decreasing heads Hysteresis effects appear to be very small, much less than the s-shaped trend of the data, which apparently is caused by the spring. The width of spring was varied in the model, as well as the amount of styrofoam, until the displacement Y was 0.75 inches at h = 2 inches, and was zero at h = 8 inches. The s-shaped trend in the data may be explained by the spring backing being incorrect to the extent that the spring was too stiff for the head range 2 to 5 inches thereby holding the disc higher in the throat and increasing the flow area slightly. Also the spring was not quite as stiff as required by the theory for the range 5 to 7.5 inches and permitted the disc to be lower than required by theory, thereby decreasing the discharge.

Figure 21 is plotted with log $\overline{Q}/Q_{min}$ as ordinate so that the same relative percentage error scale occurs for all settings, and so that the average discharge lines would be equally spaced. A change of three turns represents a displacement Δz of 3/13 inch.

Log $\overline{Q}/Q_{min}$ is plotted against number of turns in Fig. 23, where $\overline{Q}$ is the average of the discharges for a given setting. The solid line is determined by least squares and yields the equation

$$\frac{\overline{Q}}{Q_{min}} = 0.994\, e^{0.06704t} \tag{62}$$

Comparing the exponent with $z \ln \eta/2Y$, z = 13.78t. As z = 13t by direct measurement, the discrepancy can best be explained as due to a change in discharge coefficient. Tests with a fixed disc yielded a change in discharge coefficient when the spring support stands were in place that conforms generally to the relationship z = 13.78t.

The disc was stabilized by placing the steel can around the styrofoam floats for the settings 15 to 27 turns.

low Meter. The "0 turn" position for the flow meter was near the minimum roat section for h = 2 inches. The discharge Q_{min} was taken as 20.05 g.p.m. s in the case of the flow controller. The data as plotted in Fig. 22 generally onfirms the relationship that discharge varies linearly with head for any etting. A slight s-shaped trend in the data, similar to that of the flow controller, but reversed, can be explained by the spring being too stiff for small eads and not stiff enough for the larger heads.

The slopes m = Q/h (Q in g.p.m., h in inches) for the best radial lines rough each set of observations have been computed by the method of least quares and are shown in Table II, with the maximum percentage divergence any single observation. The taking of data for the flow meter was rather ifficult due to the large volume of storage upstream from the meter. Water as admitted to the flume from a constant head tank and the level in the flume ame to equilibrium very slowly. Measurements were usually taken before eady flow was established, which could introduce minor errors in the observations. The flow meter is inherently stable and did not require any form stabilization.

It was necessary to shift the disc on its shaft between the runs 0 - 12 and 5 - 27. Unfortunately, it was not adjusted so that the change from 12 turns 15 turns was actually 3/13 inch. This is evident in the plot of log m/m_o gainst number of turns in Fig. 24, where m_o is the value of Q/h for 0 turns m_o = 4.229). By constructing the best line through the points for 0 - 12 turns, nd for 15 - 27 turns, the relations between t and z were found to be z = 12.97t nd z = 12.87t respectively, which confirm the known relation z = 13t. This dicates a substantially constant coefficient of discharge over the whole ange of tests on the flow meter.

CONCLUSIONS

Theory and design data for both the flow controller and the flow meter have een developed, with several forms of nonlinear resistance. An open channel odel was constructed using a restrained-tip cantilever spring for nonlinear esistance, and data was taken for ten settings each as a flow controller and s a flow meter. The results generally confirm the theory and point the way the construction of accurate and relatively simple control and metering quipment.

ACKNOWLEDGMENTS

The experimental portion of this project was carried out with the aid of a rant from the Faculty Research Fund of the Horace H. Rackham School of raduate Studies of the University of Michigan. The theory and design data ere developed with the aid and support of The Dole Valve Company, 1901 W. arroll Avenue, Chicago. Mr. J. S. Baker and Mr. R. B. Kaiser assisted in king data.

NOTATION

A_1, A_o = area of opening between disc and throat

a, a_1 = manometer cross-sectional areas

a_b = area of bellows

b = width of leaf spring

C_d = drag coefficient

C_q = discharge coefficient

E = modulus of elasticity

e = base of natural logarithms

F = pressure force on disc

F_o = minimum design force on disc

f = functional relation

G = submerged weight of disc assembly

g = acceleration due to gravity

h = head drop across disc

h_o = minimum design head loss across disc

I = moment of inertia of spring section about neutral axis

K_o = stiffness of spring of length ℓ

k = ratio G/F_o

ℓ = length of spring for load $F_o/2$

ℓ' = length of disc support shaft

ln = natural logarithm

M = moment

P = force on spring assembly

P' = force on single spring element

Q = discharge

R = manometer gage difference

r = radial coordinate of throat

r' = minimum throat radius

r_o = disc radius

S_m = maximum fibre stress

s, s_o, s_1 = dimensions of manometer

t = thickness of spring

W = weight

w = unit weight of fluid

x = axial coordinate of throat profile

x' = position of disc in throat

x_0 = coordinate of spring backing

$\bar{x}$ = dimensionless length x/ℓ

Y = value of y for $h = h_0$

y = position of spring

$\bar{y}$ = dimensionless quantity y_0/Y

y_0 = coordinate of spring backing

z = discharge setting

α = constant, angle

β = angular deflection

δ = displacement of single spring element from minimum design head position

ζ = dimensionless variable

η = ratio of maximum design head to minimum design head

θ = polar coordinate

ρ, ρ_0 = polar coordinate

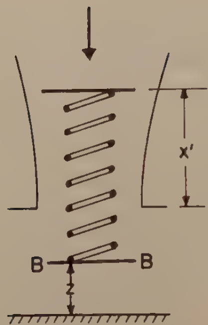

Fig. 1 Linear Spring Flow Controller

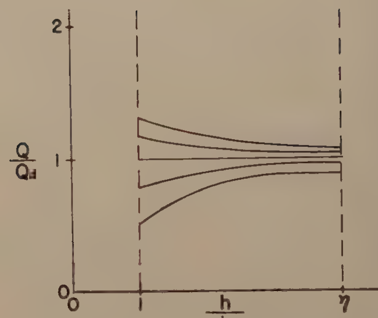

Fig. 2 Head-Discharge Curves

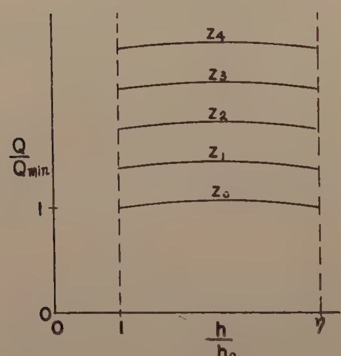

Fig. 3 Head-Discharge Curves

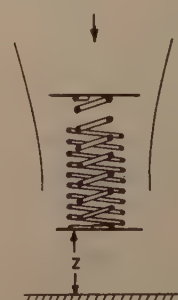

Fig. 4 Two-Spring Controller

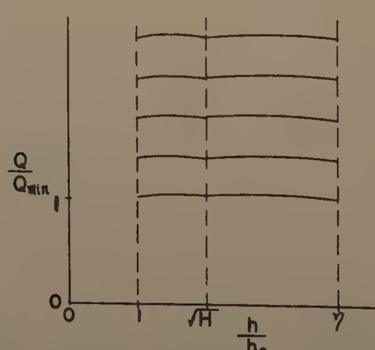

Fig. 5 Head-Discharge Curves

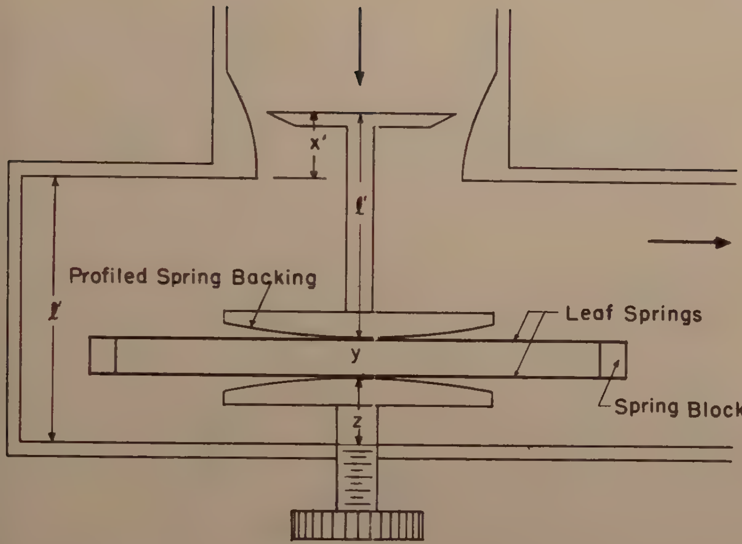

Fig. 6 Nonlinear Spring Flow Controller

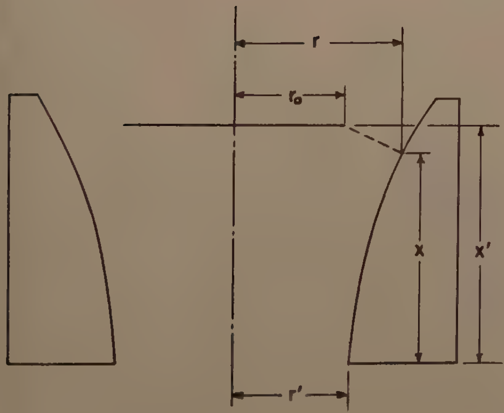

Fig. 7 Notation for Disc and Throat

Fig. 8 Envelope Solution for Throat Profile

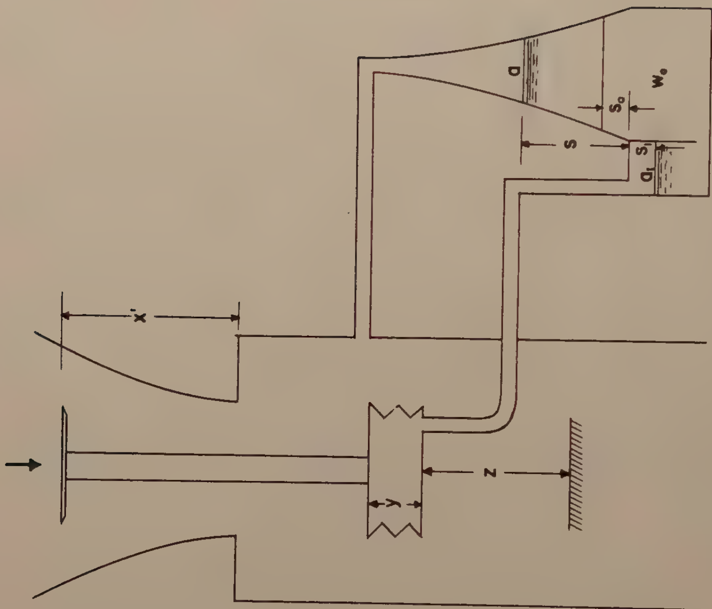

Fig. 10 Flow Controller with Liquid Spring

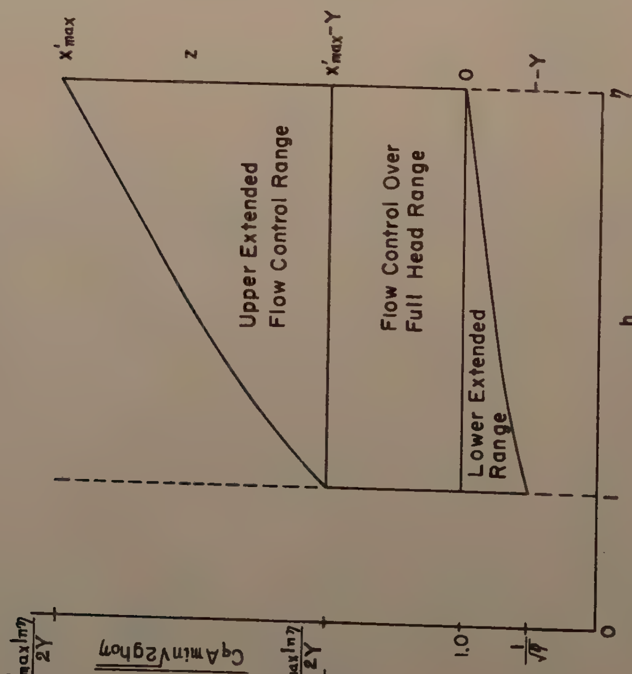

Fig. 9 Total Flow Control Range

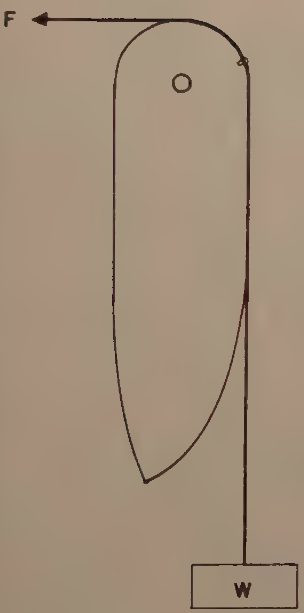

Fig. 11 Profiled Cam

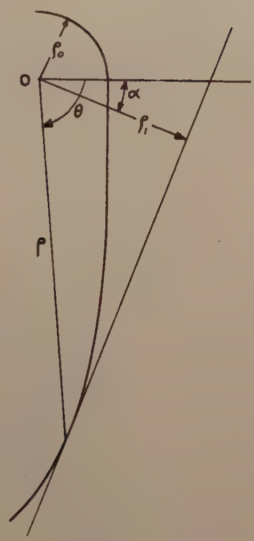

Fig. 12 Notation for Cam Equation

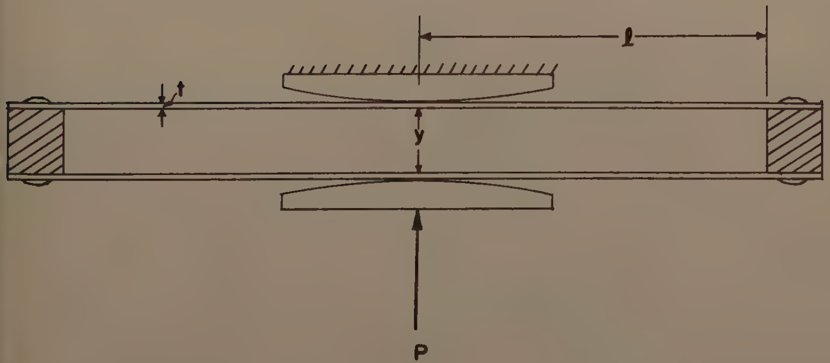

Fig. 13 Restrained Tip Cantilever Spring

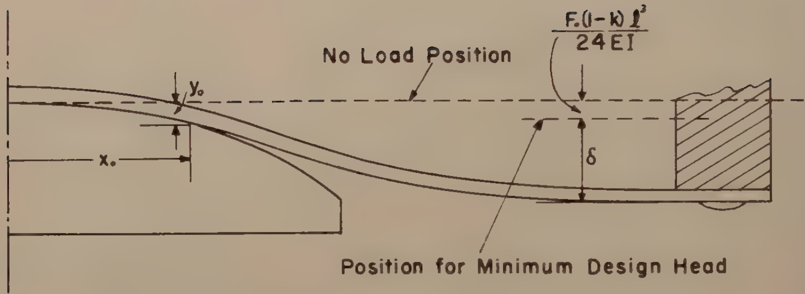

Fig. 14 Notation for Cantilever Spring

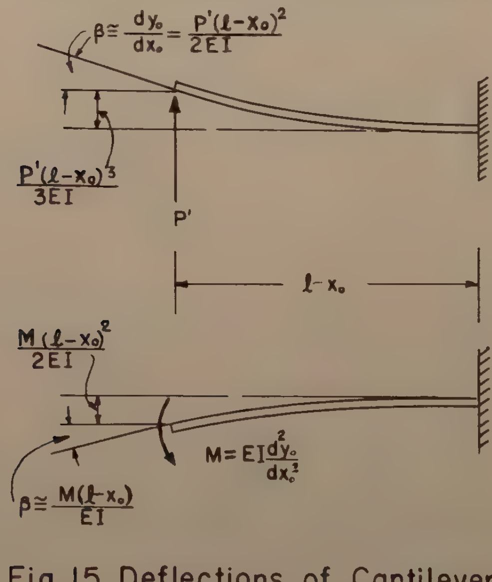

Fig. 15 Deflections of Cantilever Spring

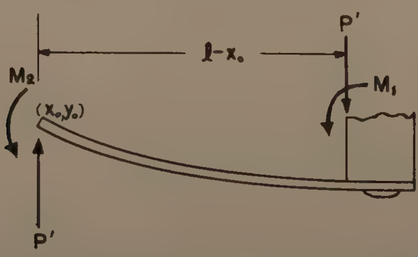

Fig. 16 Free body Diagram of Spring

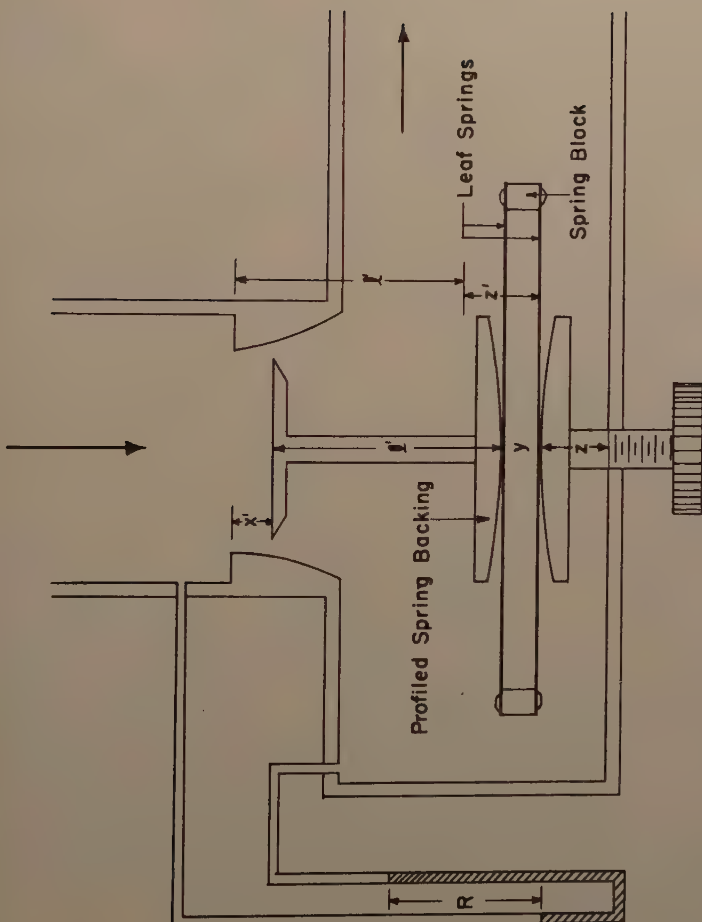

Fig.17 Nonlinear Spring Flow Meter

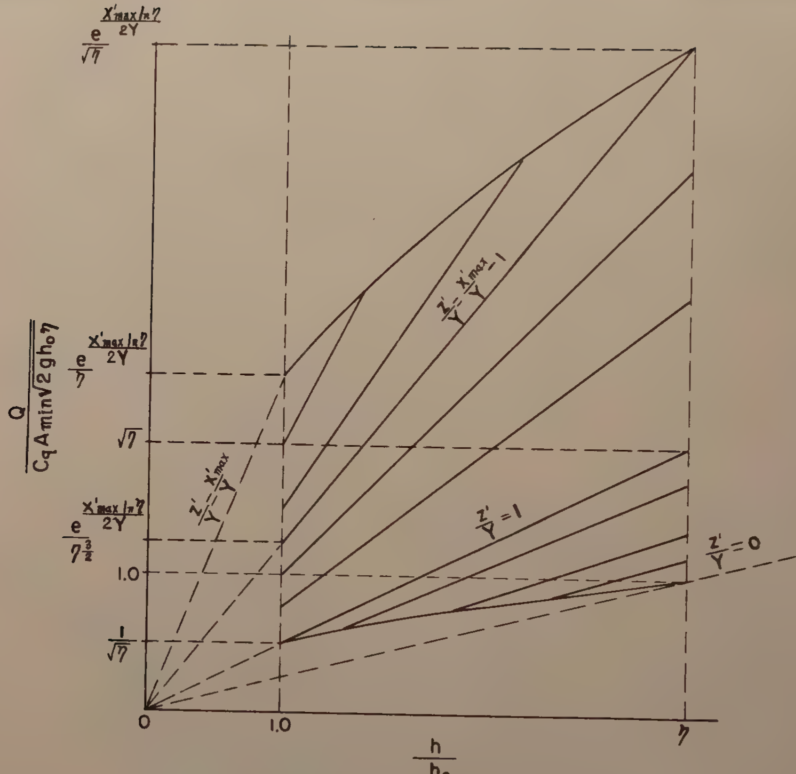

Fig. 18 Flow Meter Range

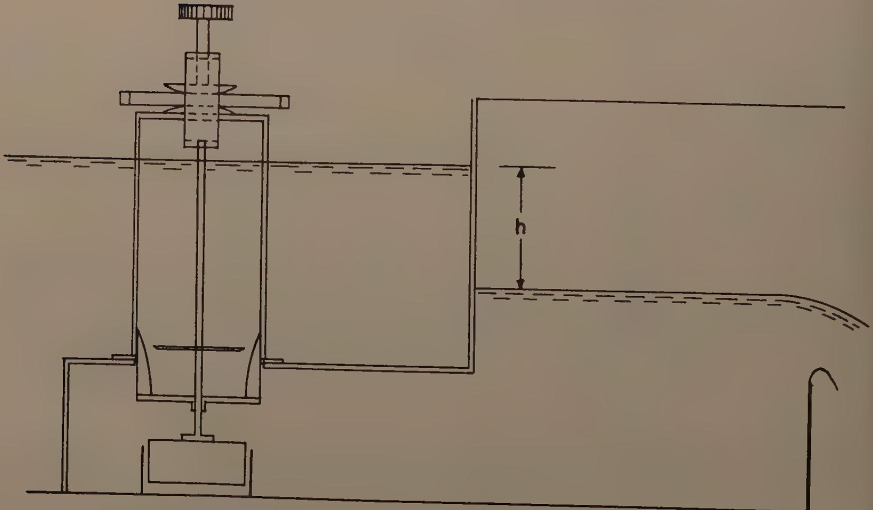

Fig. 19 Elevation Section of Controller

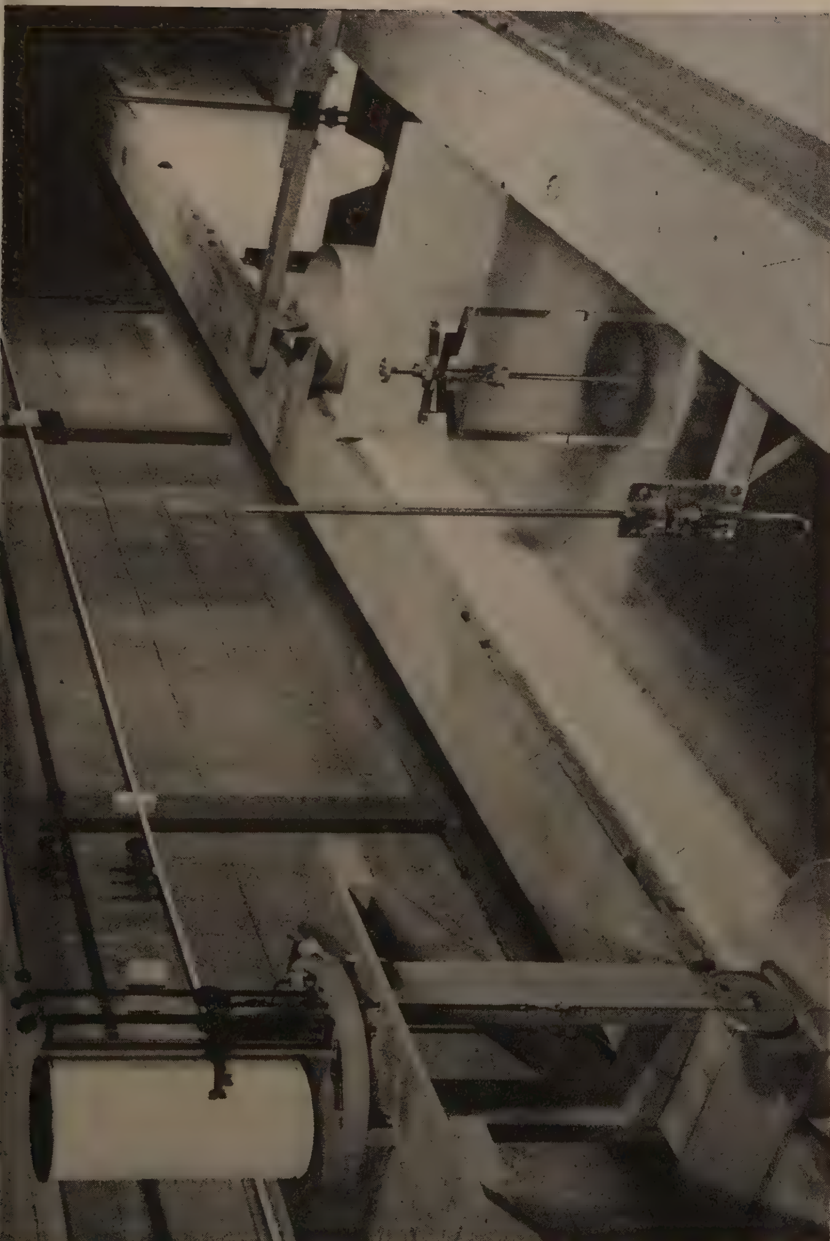

Fig. 20 Flow Controller in Place

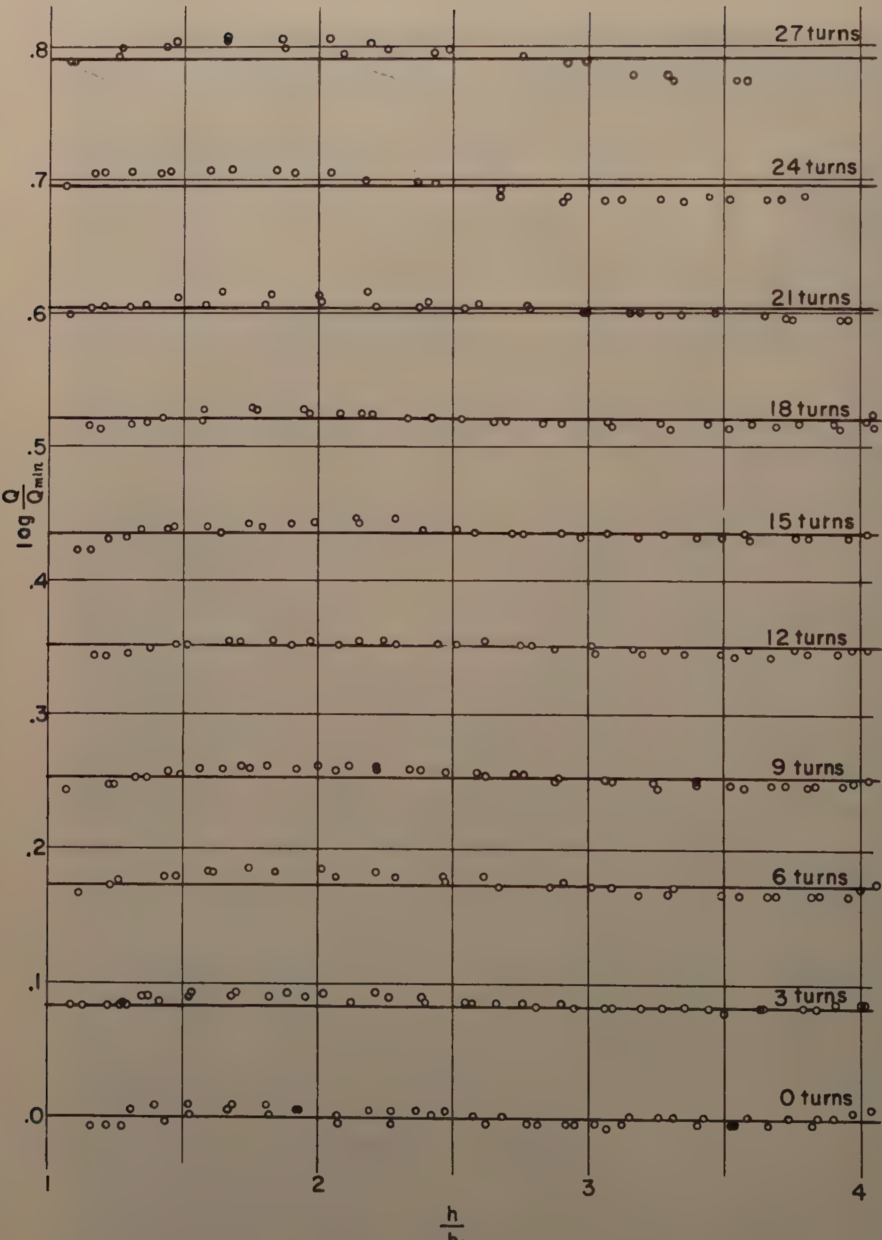

Fig. 21 Flow Controller Test Data

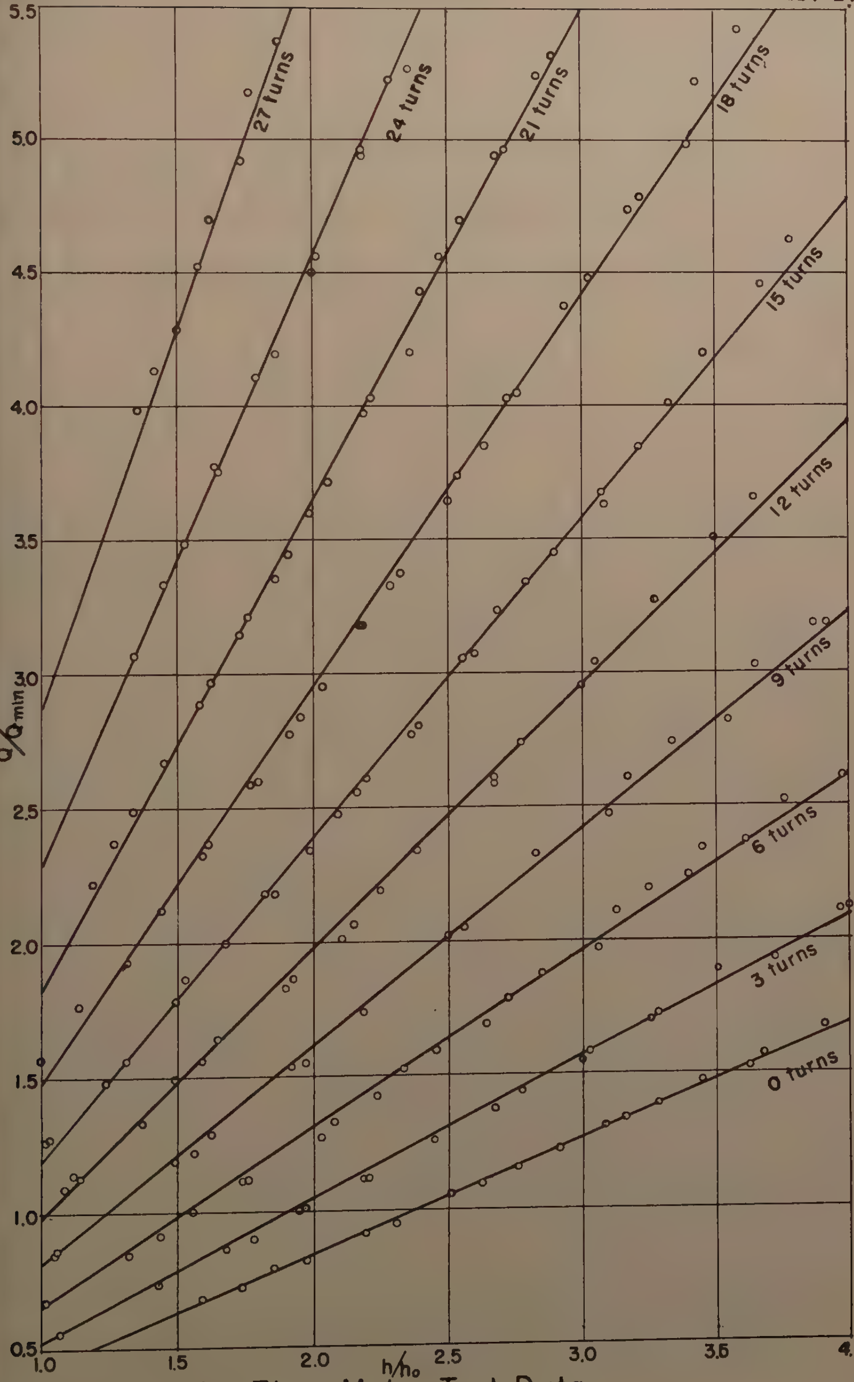

Fig. 22 Flow Meter Test Data

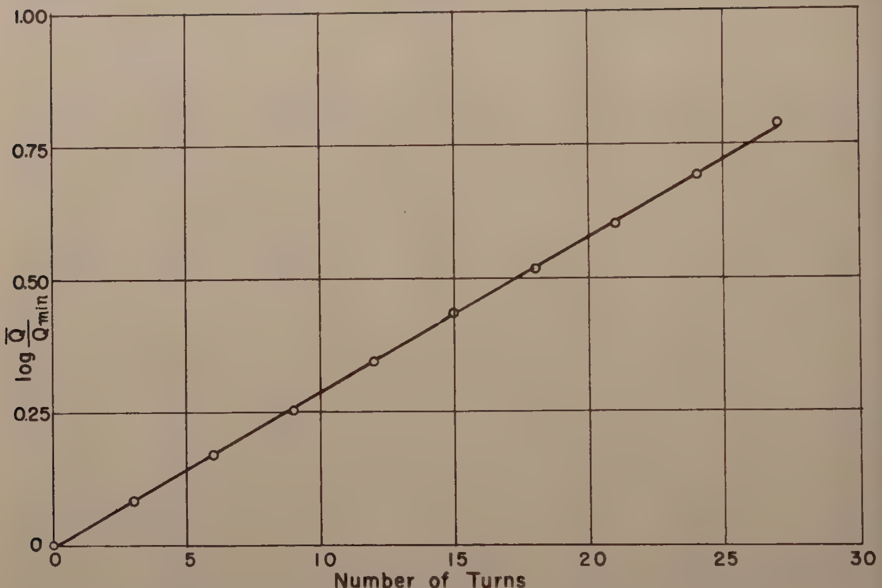

Fig. 23 Discharge Related to Number of Turns — Flow Controller

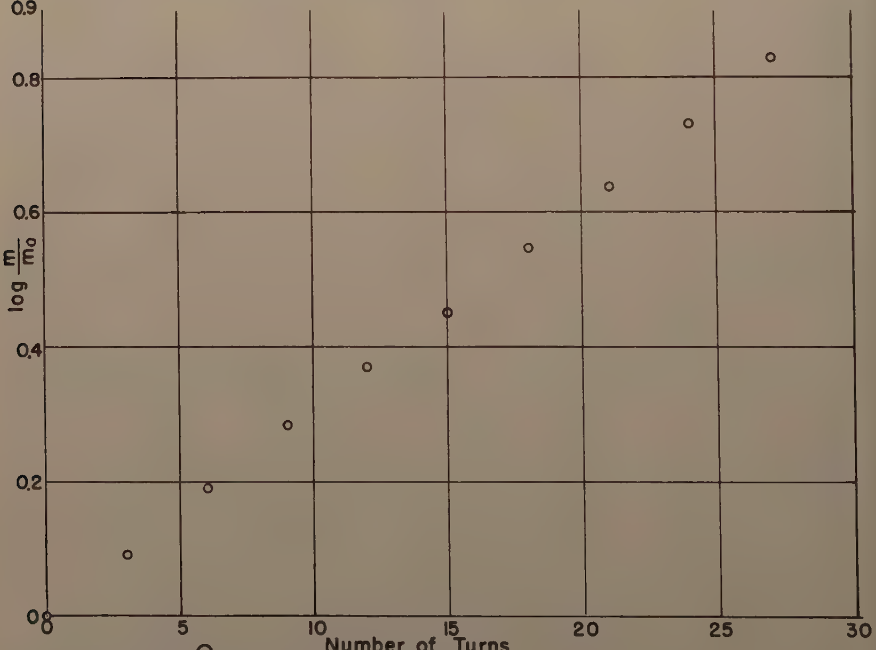

Fig. 24 $\frac{Q}{h}$ as a Function of Number of Turns — Flow Meter

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SEVEN EXPLORATORY STUDIES IN HYDRAULICS

Hunter Rouse,[1] M. ASCE
(Proc. Paper 1038)

FOREWORD

Immediately following the Sixth Hydraulics Conference, held at Iowa City in June of 1955, the Iowa Institute of Hydraulic Research conducted an intensive three-week course in experimental techniques, in which twenty-one graduate engineers from various organizations participated. The purpose of the course was not only to acquaint the class with new types of instrumentation, but also to instill in them an appreciation for the methods of research.

During the first part of the course, eight of the elementary experiments in mechanics of fluids taught at Iowa to engineering students were carried out and reported upon by the class in teams of three. These experiments included curvilinear flow with cavitation in a closed conduit; curvilinear flow with a free surface; viscous flow in a pipe; turbulent flow in smooth and rough pipes; form effects at conduit transitions; diffusion of a submerged jet; fall velocity of spheres; and flow past a circular cylinder. Messrs. D. W. Appel, P. G. Hubbard, E. M. Laursen, and D. E. Metzler of the Institute staff acted as instructors during this introductory period.

The second part of the course involved eight experiments utilizing advanced methods of measurement: the Pitot cylinder for constructing a two-dimensional flow pattern; resistance wires, pressure cell, and Sanborn recorder for wave studies; the electrolytic tank for pressure distribution at a three-dimensional inlet; an electrical analog computer for the cross-thrust on neighboring cylinders; the hot-wire anemometer for submerged-jet turbulence; stagnation-tube surveys for boundary-layer shear on a plate; multiple-resistance gage for scour at a bridge pier; and a strain-gage dynamometer for lift and drag on a foil. The foregoing instructors were assisted in these experiments by Messrs. M. Hug, S. C. Ling, T. T. Siao, and A. Toch.

In the third part of the course, which is of primary concern herein, the members of the class were encouraged to undertake short investigations of an original nature, to the end of determining new and publishable information through application of the foregoing techniques. The following seven topics

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were suggested:

Development of the non-circulatory waterspout
Penetration of a jet into a counterflow
Surface profiles at a submerged overfall
Coefficient of contraction for a submerged jet
Maximum amplitude of oscillation of a cylinder due to its vortex wake
Deflection of a jet by a normal wall
Influence of slope and roughness on the free overfall

Experimental facilities for the investigations were improvised from equipment at hand, and a member of the Institute staff was assigned for consultation to each of the seven teams. In the short time available, each team performed the required experimental measurements and computations, and six of them submitted preliminary reports to serve as a record for reference purposes. The seventh team, under the leadership of Mr. J. C. I. Dooge, was able to give somewhat more time to its project, and the report that he finally submitted was well beyond the preliminary stage. Such amplification of the other reports as proved necessary to prepare them for publication in the following form was carried out subsequently by the advisers under the writer's editorship, and the illustrations were brought to their present state of uniformity by Mr. Metzler. Although none of the articles even approaches the final word upon the subject, it is believed that each will provide both useful information and an incentive to further research.

DEVELOPMENT OF THE NON-CIRCULATORY WATERSPOUT

J. Davidian,[2] J. M. ASCE, and J. E. Glover[3]
(D. W. Appel,[4] J. M. ASCE, Adviser)

Statement of the Problem

The most familiar occurrences of waterspouts in nature are those accom-anying cyclonic storms over the oceans. A vortex is formed by the inter-ction of air masses, into the low-pressure core of which water is drawn, ometimes to great heights. The circulatory motion of the vortex is evident-' an essential part of the phenomenon. Less well known is the non-ırculatory waterspout, which may form in the low-pressure zone produced y radial flow toward an intake in either of two fluid layers of different den-ity. In plumbing systems the phenomenon is known as back-siphonage, ater as well as air being drawn into the unsubmerged end of an open supply pe that is momentarily under vacuum. The purpose of this study was to etermine experimentally the rate of flow necessary to develop a waterspout ıst to the point at which the second as well as the first fluid is carried into e pipe. This represents the critical limit for safeguarding water-supply nes from contamination, and for the general separation of liquids of nearly e same density by decantation.

The geometric variables in the problem are the diameter D of the suction pe and the vertical distance y from its open end to the interface between e fluids. If one assumes for the purpose of a preliminary analysis that the fects of viscosity and surface tension are negligible, the only significant uid properties are the density of the upper (or moving) fluid ρ and the dif-rence between the specific weights of the two fluids $\Delta\gamma$. As the flow vari-ble, the mean inlet velocity V at which a waterspout forms has been cho-en. When these variables are combined by using the techniques of mensional analysis, the two dimensionless parameters obtained are y/D d $\rho V^2/\Delta\gamma D$. The first of these is a geometric variable, whereas the latter epresents the ratio of inertial and gravitational forces and hence is a form the Froude number. This analysis indicates that a single series of mea-ırements of the critical velocity for the formation of a waterspout over the actical range of relative heights of the suction pipe should suffice to deter-ine the function

$$\frac{y}{D} = \varphi\left(\frac{\rho V^2}{\Delta\gamma D}\right)$$

owever, to check the validity of the assumptions made in the analysis and to

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Research Engineer, Iowa Institute of Hydraulic Research, State University of Iowa, Iowa City.

demonstrate the generality of the results, two series of tests were conducte In the first series a waterspout was formed in water under air, and in the second the waterspout was formed in saline water under fresh.

The Experiments

To simplify observation and extend the range of the first series of tests, three suction pipes 0.5, 1.5, and 3.0 inches in nominal inside diameter were used interchangeably. Each of these in turn was mounted vertically above a open tank of water 3 feet in diameter and 3 feet deep in which the elevation the water surface could be regulated by means of supply and drain lines and measured by a hook gage (see Fig. 1). The inflow of air necessary to form the waterspout was produced by connecting the pipe to a large tank from which the air had been exhausted to a pressure of 10 inches of mercury abso lute.

As shown in the figure, a differential manometer was connected between stagnation tube and a wall piezometer in the horizontal portion of the suctio pipe for measurement of the centerline velocity at the gaging section. This was related to the mean velocity $V = Q/\frac{1}{4}\pi D^2$ at the inlet section by means of the Bernoulli equation, through the assumption of adiabatic expansion between the inlet and the gaging section. The pressure at the inlet was read o a barometer and the pressure at the gaging section was obtained from a Bourdon gage. The coefficient of the flow meter was determined by calibration with water.

With the water surface set at the desired elevation for a test, the control valve was slowly and continuously opened until the waterspout that develope just began to carry water into the pipe. At this instant the two gages were read at the flow-measuring section. The value in the suction line was then closed, the vacuum tank pumped down, and a different water level establishe in preparation for another test.

In the second series of experiments a glass-walled tank 1 square foot in cross section and 2 feet high was used. This permitted visual observation of the interface between clear fresh water above and dyed salt water (specif gravity = 1.1) below. The arrangement of the suction pipe and vacuum tank was the same as before. The rate of flow, however, was determined by tim the fall of the free surface in the tank, which was quite feasible with the low rates required. Because of the limited size of the tank, only the 0.5-inch a 1.5-inch suction pipes were used.

The Results

All experimental data are presented graphically in Fig. 2 in the dimensi less form suggested by the preliminary analysis. It can be seen that the te using different sizes of pipe as well as entirely different fluids produced re markably consistent results. Some scatter was to be expected, particularl for the 1/2-inch pipe in the two-liquid test, because of the difficulty in regu lating the rate of flow to obtain critical conditions. The tendency of the poi to follow a single curve (approximated by the power relationship $y/D = 0.42\left[V/\sqrt{D\Delta\gamma/\rho}\,\right]^{\frac{1}{2}}$) indicates that the assumption of negligible visco and capillary effects was justified for such an exploratory study. A

hotograph of the waterspout in air is shown in Fig. 3, and one of the saline aterspout in fresh water is presented for comparison in Fig. 4.

In conclusion, the limitations on the applicability of these results should e emphasized. The tests were run with fluids of low viscosity in relatively rge tanks. Ripples, pendulations in the level of the interface, and the proxnity of walls or other solid obstacles can be expected to increase the gap at is necessary to prevent formation of a waterspout. Thus, the results resented represent the minimum spacing required. It is evident that the alue of y/D of 2 sometimes used in design is not sufficiently great to inure against the formation of a waterspout at high rates of siphonage.

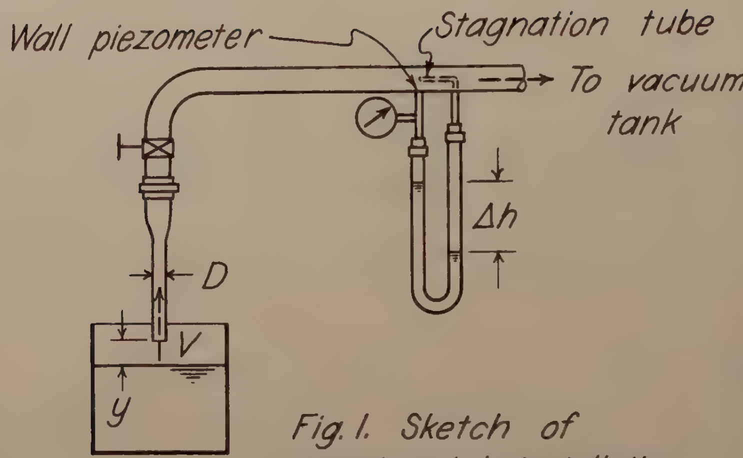

Fig. 1. Sketch of experimental installation

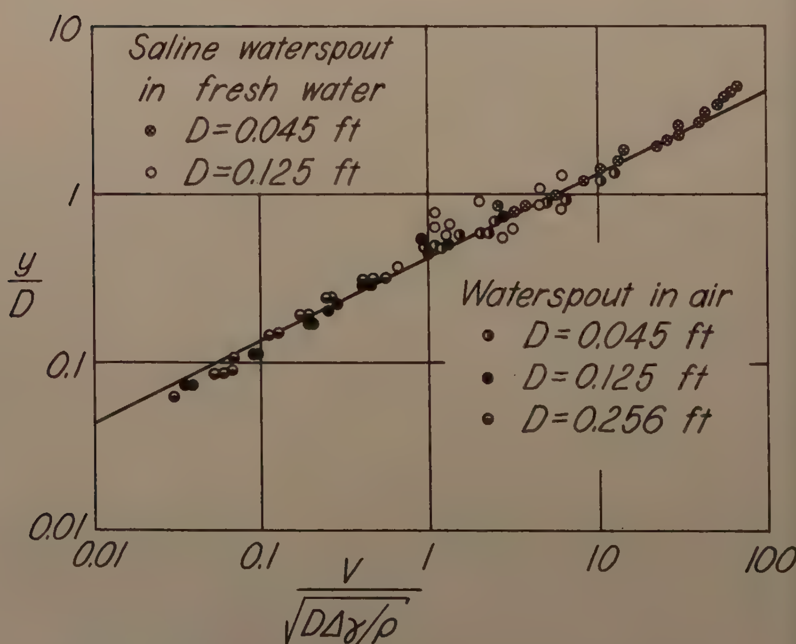

Fig. 2. Composite plot of measured data

Fig. 3. Surface profile of waterspout in air.

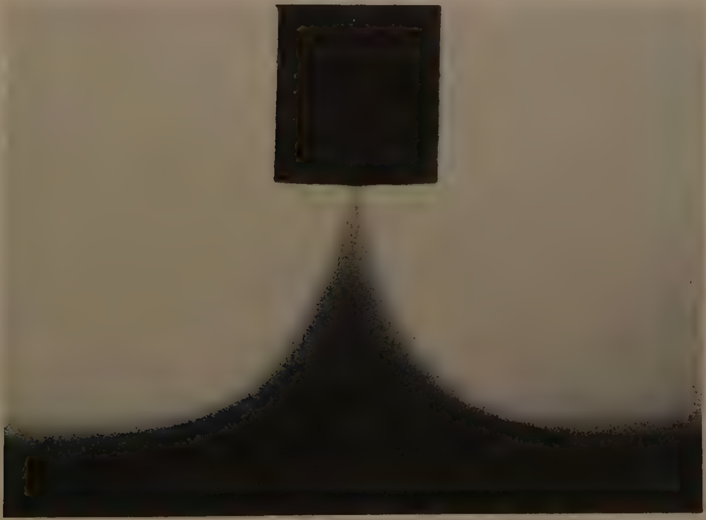

Fig. 4. Saline waterspout (dyed) in fresh water.

PENETRATION OF A JET INTO A COUNTERFLOW

J. Arendt,[5] H. A. Babcock,[6] and J. C. Schuster[7]
(H. Rouse,[1] M. ASCE, Adviser)

If a fluid jet is directed upstream against a uniform flow of the same fluid, it is to be presumed that the jet will penetrate some distance into the oncoming flow and then be turned back upon itself and carried downstream, as shown schematically in Fig. 5. The degree of penetration—in fact, the geometry and kinematics of the entire flow pattern—could be expected to depend as a first approximation upon the diameter and velocity of the jet and the velocity of the counterflow; that is,

$$f_1(x, r, d, v, v_j, v_0) = 0$$

which may be reduced to the non-dimensional form

$$\varphi_1\left(\frac{x}{d}, \frac{r}{d}, \frac{v}{v_0}, \frac{v_j}{v_0}\right) = 0$$

According to this relationship, two jets will yield similar patterns of flow (i.e., similar distributions of v/v_0) only if the ratio v_j/v_0 is the same for both, under which circumstances the distance upstream to the point of stagnation (or the diameter of the enclosed jet flow, or any other characteristic length) will be a specific number of jet diameters. However, available information on submerged jets[8] would lead one to conclude that approximate similarity may be attained for jets of sufficiently small diameter and high velocity through the control of a single jet variable rather than two together. This variable is the momentum flux:

$$M = Q_j \rho v_j = \tfrac{1}{4}\pi d^2 \rho v_j^2$$

Under these circumstances,

$$f_2(x, r, v, v_0, \rho, M) = 0$$

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whence, in non-dimensional terms,

$$\varphi_2\left(\frac{x}{m}, \frac{r}{m}, \frac{v}{v_0}\right) = 0$$

in which $m = \sqrt{M/\rho}\,/v_0$.

Experiments to permit an evaluation of this functional relationship were performed in an air tunnel having a 3x3-foot cross section and a velocity range from 15 to 45 feet per second. Measuring equipment consisted of a differential water manometer read by vernier to 0.001 foot, a hypodermic-needle stagnation tube, and calibrated Pitot cylinders with pairs of holes at 30° and 180° from each other for determining the velocity magnitude and direction. The jet was produced by discharging compressed air from a copper tube of 0.015-foot inside diameter mounted at the axis of the tunnel test section approximately midway along its 12-foot length.

Preliminary observations of the flow pattern were made by means of threads giving a visual indication of the distance that the jet penetrated into the counterflow and the radial extent of the zone of mixing. Once a sufficient length of penetration relative to the initial diameter of the jet had been established, measurements of the velocity distribution were made at a series of cross sections. Whereas a proper investigation would require the precise determination of both the magnitude and the direction of the velocity vector, in the limited time available it was possible to obtain only the magnitude. Even this, however, permitted an approximate evaluation of the flow pattern by the following technique.

Assuming as a first approximation that $v_x \approx v$, diagrams are prepared of $2\pi r\, v_x$ as a function of r , as in Fig. 6. Since $Q = 2\pi\int r\, v_x\, dr$, use of a planimeter then permits the preparation of diagrams for Q as a function of r . Now a stream tube (which is formed of streamlines) represents by definition a surface through which no fluid passes. It follows that Q must be the same at successive cross sections of the same stream tube, and as well that the incremental rate of flow ΔQ between coaxial tubes must also be constant. It is hence possible to approximate the location of streamlines in an axial plane by noting the values of r for the same flow rates 0 , ΔQ , $2\Delta Q$, etc., at successive longitudinal distances x . From the direction of flow indicated by these streamlines, a second approximation may be obtained, which is usually sufficient.

The experimental results were evaluated in this general fashion, the sole difference being that non-dimensional rather than dimensional notation was used throughout. In other words, the quantity $2\pi r v/m v_0$ was integrated with respect to r/m to yield $Q/m^2 v_0$, and for the construction of streamlines the increment $\Delta Q/m^2 v_0 = 0.2$ was chosen. The resulting streamline configuration is shown in Fig. 7. This may be presumed to indicate, at least as a first approximation, the pattern of flow that would be produced by any jet of high enough relative velocity and small enough diameter for the latter to be inappreciable in comparison with the length of penetration (i.e., x_s/d must be greater than perhaps //25). Under such circumstances, it follows that, since $x_s/m = -2.7$, $m = \sqrt{M/\rho}\,/v_0$, and $M = \frac{1}{4}\pi d^2 \rho\, v_j^2$,

$$\frac{x_s}{d} = -2.7\sqrt{\frac{\pi}{4}}\,\frac{v_j}{v_0}$$

Although the measurements leading to the pattern shown in Fig. 7, lacking as they were in both precision and completeness, must be regarded as exploratory only, they point at once to fruitful directions of more extended study. Three are readily apparent: first, verification of the assumption that m is a widely useful characteristic of jet performance by tests under a considerable range of conditions; second, determination of the pattern of secondary flow—i.e., turbulence—in the same generalized manner; third, evaluation of the effect of viscosity in terms of a Reynolds number such as $v_0 m/\nu$, and perhaps eventually a completely analytic study of the pattern for the limiting case of laminar flow.

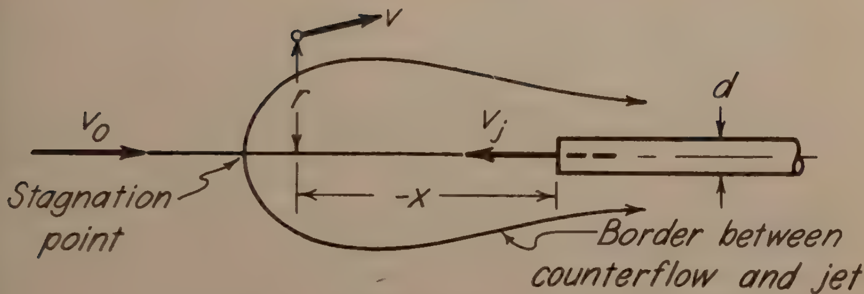

Fig. 5. Definition sketch

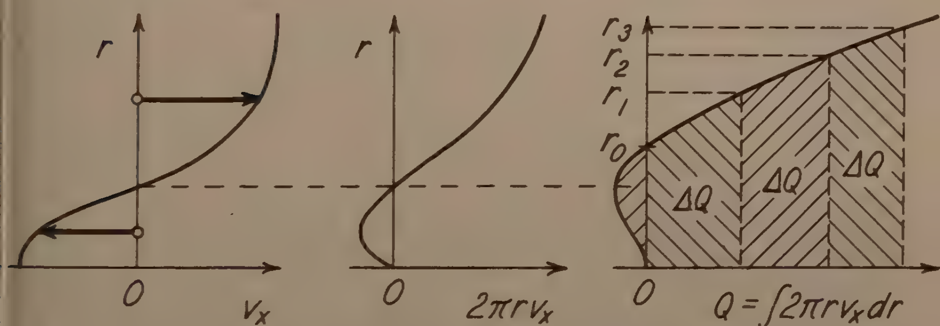

Fig. 6. Determination of streamline locations

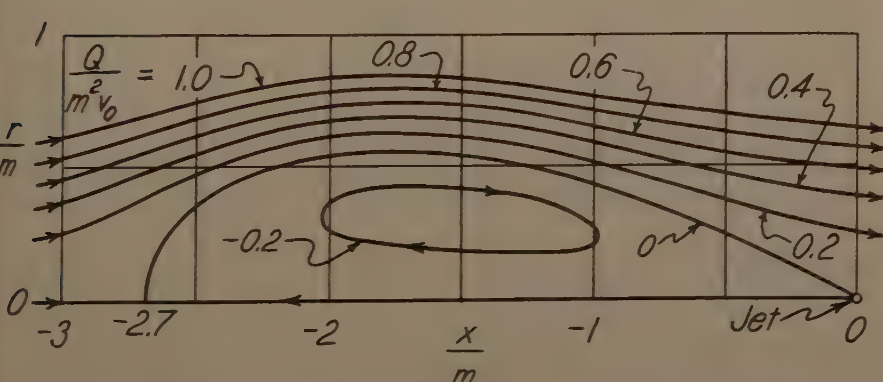

Fig. 7. Generalized representation of the flow pattern

SURFACE PROFILES AT A SUBMERGED OVERFALL

L. F. Ingram,[9] R. E. Oltman,[10] A. M. ASCE, and H. J. Tracy[11]
(E. M. Laursen,[12] A. M. ASCE, Adviser)

Depending upon the interrelationship of a number of variables, the free surface at a submerged overfall will assume one of several basically different forms. In addition to the coordinate position, the pertinent variables include the velocity and depth of the oncoming flow; the depth of the tailwater in the forebay; the channel slope, roughness, and cross-sectional dimensions; and the dimensions of the forebay. For the purpose of this exploratory study, the channel was made smooth and horizontal and the drop as great as possible, changes in boundary alignment were limited to the vertical, and the surface configuration was observed in the crest vicinity only. The function under consideration was thus reduced to one between the following major parameters:

$$\frac{y}{y_0} = \varphi\left(\frac{x}{y_0}, \frac{y_t}{y_0}, F\right)$$

Herein x and y represent horizontal and vertical distances from the overfall crest, y_0 and y_t are the initial and tailwater depths, and F is the Froude number $V/\sqrt{gy_0}$.

The experimental investigation of this function was conducted in a glass-walled flume 1 foot wide, 2 feet deep, and 10 feet long. Falsework in the flume created a 1-foot drop, the horizontal portion of the channel being 2.5 feet long and having a quarter-circle transition at its upper end. Discharge through the flume was measured by a calibrated elbow meter and an air-water manometer. The depth of the approaching flow could be controlled by a sluice gate at the beginning of the horizontal channel and measured by a point gage at a section 0.6 foot (i.e., $4y_0$) upstream from the drop. The depth of the tailwater was controlled by vertical slats at the end of the flume, 7.5 feet beyond the drop, and was measured by floor piezometers connected to open manometer tubes. The water-surface profile was measured with a point gage graduated to 0.001 foot and mounted on a carriage sliding on level stainless-steel rails.

The first step in the standard operating procedure was to adjust the control valve and sluice gate until the desired rate of discharge and depth of approach were obtained. In all runs the approach depth was maintained at 0.15 foot,

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and Froude numbers of 1, 2, 3, and 4 characterized the different series of runs. In each series successively greater tailwaters were established and the water-surface profiles measured. Except for the maximum tailwater, values of y_t/y_0 which were either positive or negative integers were chosen. The maximum tailwater used was that which formed a hydraulic jump just upstream from the drop; any slight increase in the tailwater above this value resulted in submergence of the sluice gate. Observations were also made of the tailwater elevation corresponding to the transitions between a plunging and a riding nappe and between a riding nappe and a hydraulic jump; because of something akin to a hysterisis effect, these observations were made with both a rising and falling tailwater.

The water-surface profiles as measured are presented in dimensionless coordinates in Fig. 8. A more graphic representation of the typical flow patterns can be obtained from Fig. 9, wherein photographs of the four runs with the highest tailwaters at $F = 2$ in Fig. 8 are shown. Three distinct regimes of flow can be seen. At the low tailwaters the nappe plunges below the surface. The tailwater elevation in this regime of flow affects the nappe principally insofar as it determines the pressure under the nappe: the lower the tailwater, the more the nappe is depressed. Above a certain tailwater elevation, depending on the Froude number, a distinctly different flow pattern obtains in which the nappe does not plunge, but rides the surface downstream from the drop. As the tailwater elevation is further increased, the surface waves which characterize this regime of flow become progressively larger until the leading wave begins to break. Above this second critical tailwater elevation, a hydraulic jump forms with its toe slightly upstream from the drop. A small increase in the tailwater elevation is then sufficient to cause the jump to move entirely into the channel. For Froude numbers appreciably less than 2 the jump is of the undular rather than the breaking type, and the upper regime becomes indistinct from the intermediate. At a Froude number approaching unity, at which limit the sluice gate could no longer be used as a control, it is obvious that even the undular form of jump could not occur; tailwater elevations appreciably above the line of critical depth then resulted in complete submergence of the overfall and consequently in flow with a Froude number less than unity.

The relative tailwater elevations at which the transitions between flow regimes occurred are shown in Fig. 10 as functions of the Froude number. Although no instability of the nappe was noted, there is evidently a significant difference between the values for rising and for falling tailwater. Because the flow tended to be stable, the critical elevations for falling tailwater are invariably below those for rising tailwater.

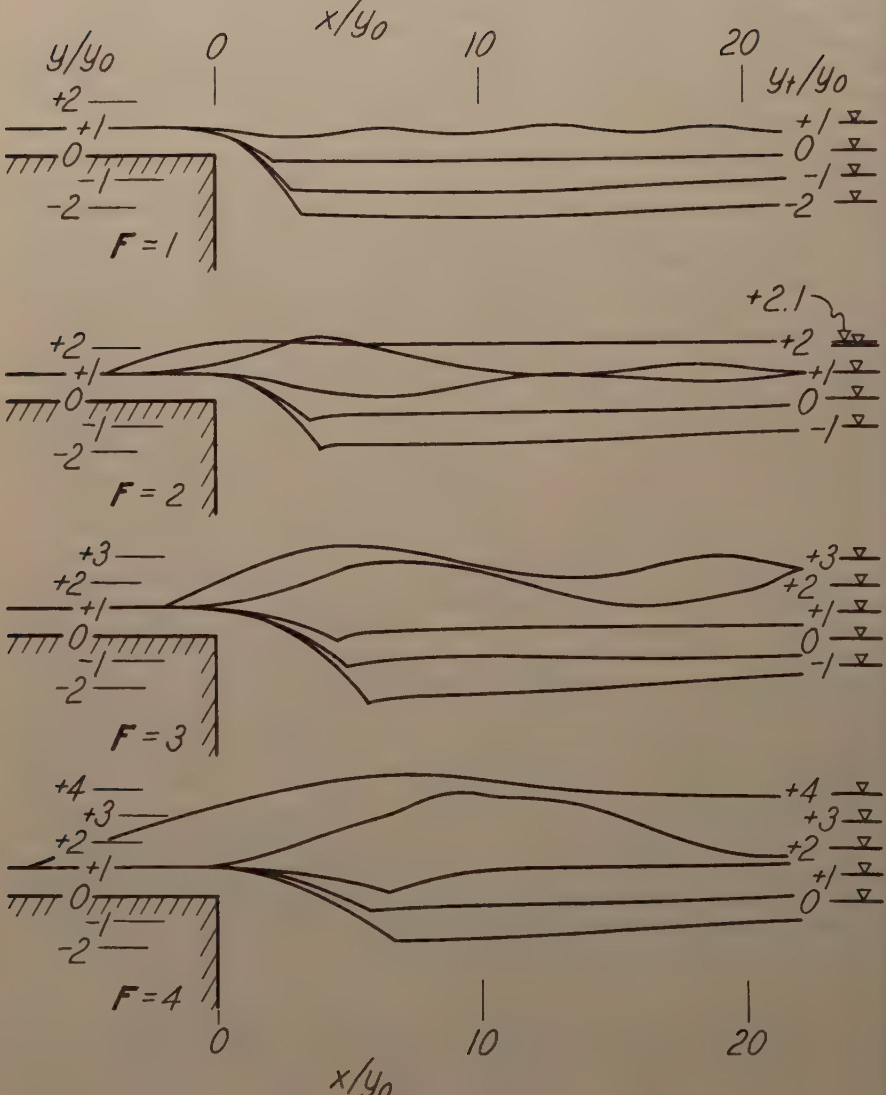

Fig 8. Measured variation in surface profile with Froude number and depth of tailwater

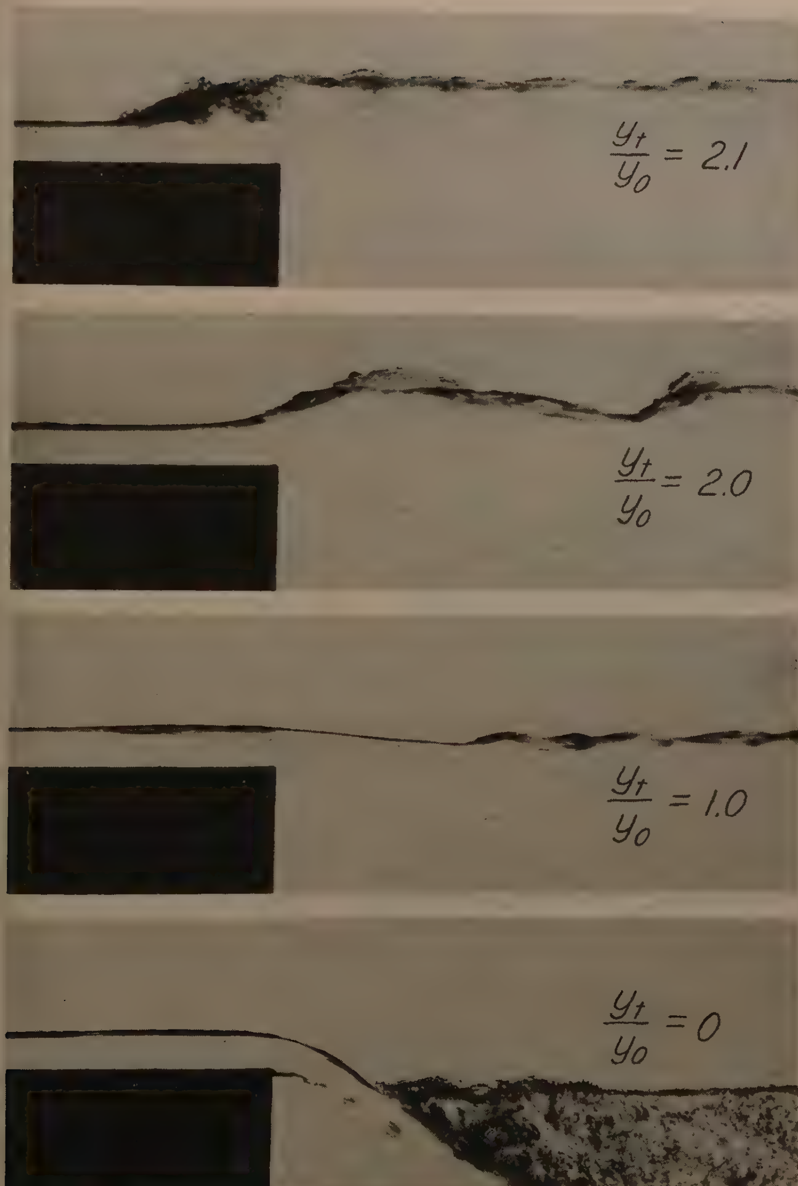

Fig. 9. Photographs of surface profiles for F = 2.

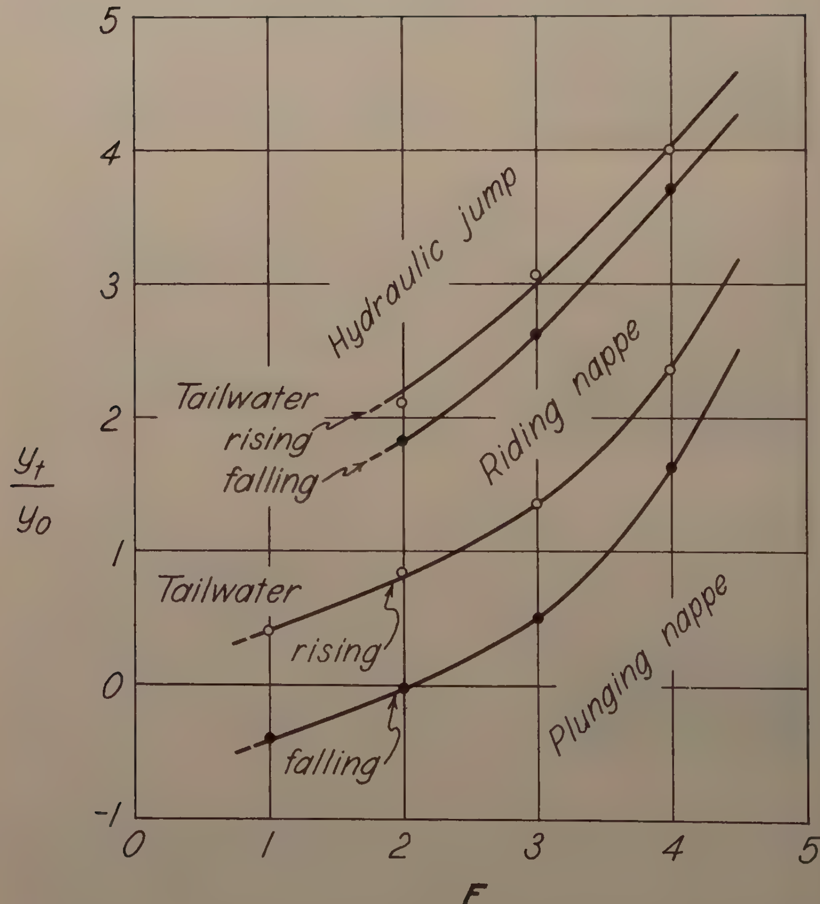

Fig. 10. Lines of demarcation between regimes of flow

COEFFICIENT OF CONTRACTION FOR A SUBMERGED JET

D. D. Curtis,[13] M. ASCE, J. E. Martinez,[14] J. M. ASCE, and V. Vazquez[15]
(H. Rouse,[1] M. ASCE, Adviser)

In computing the rate of discharge for a submerged orifice, it is customary to assume that the coefficient of jet contraction is the same as it would be if the jet were free. This assumption apparently yields satisfactory results, else it would not have been made so consistently for the past century or more. It is not conceivable, however, that the flow patterns of the free and submerged jets would be identical or even closely similar. The present experiments were hence undertaken to compare both the geometric and the kinematic characteristics of a free and a submerged jet issuing from the same orifice under the same differential head.

The tests were conducted in a glass-walled flume having a length of 25 feet, a width of 2.5 feet, and a depth of 3 feet. A 2-inch sharp-edged orifice was placed 1.25 feet above the floor of the flume in the middle of a vertical steel plate separating the flume from a 6-foot stilling tank at its upstream end. Closure of the flume at its downstream end by a plate 2 feet high provided not only a skimming weir for maintaining a constant depth of submergence but also a volumetric tank for preliminary determination of the head-discharge relationships. A gage carriage on rails, point and hook gages reading by vernier to 0.001 foot, a differential manometer with the same precision of indication, a hypodermic-needle stagnation tube, and a stop watch were the measuring instruments used.

Figure 11 shows the results of the calibration runs for both the free and the submerged jet, from which it will be seen that essentially the same coefficient of discharge—and hence essentially the same coefficient of contraction—applies to both cases under the same differential head. The actual contraction of the free jet under a head of 2 feet was determined by measuring the surface profile in the vertical plane, computing the elevation of the jet axis therefrom, and eliminating the effect of gravitational deflection by subtracting from the measured profile elevations the corresponding deviation of the axis from the horizontal. The superposed results are shown in non-dimensional form in Fig. 12, together with computed values for the case of irrotational flow.[16] Although the profiles differ somewhat in form, the limiting ordinates are very nearly the same, from which it is found that $C_c = (0.78)^2 = 0.61$.

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15. Assistant Professor, Monterrey Institute of Technology, Monterrey, N. L., Mexico.
16. Rouse, H., and Abul-Fetouh, A. H., "Characteristics of Irrotational Flow through Axially Symmetric Orifices," Journal of Applied Mechanics, Vol. 17, No. 4, 1950.

From the stagnation-tube traverses of the submerged jet a family of velocity-distribution curves, selected ones of which are shown in Fig. 13, was obtained. From this sequence it is seen that the shear around the periphery of the jet both decelerates the flow within the jet and brings into motion the fluid immediately surrounding it; the region in which this effect occurs necessarily expands steadily with distance from the orifice. The extent of the jet contraction must, as a result, be defined rather arbitrarily. Now the most logical designation of the jet profile is that within which the rate of flow is the same at successive sections. The quantity $2\pi rv$ at each section was hence plotted against the radius r, the integral $Q = 2\pi \int r\, v\, dr$ of the area enclosed by each curve thereafter being plotted against r. From each of these curves the magnitude of r for which Q equals Q_0, the rate of orifice discharge, could readily be determined. A series of the resulting values will be seen in Fig. 14, together with the profile of the free jet from Fig. 12.

Comparison of the trend of the points with that of the curve will show that the profile of the submerged jet attains minimum proportions approximately a half diameter from the orifice but then gradually expands, whereas the free jet—although it theoretically approaches its limiting diameter asymptotically—actually undergoes little further change beyond a half diameter from the orifice. The minimum dimension of the submerged-jet profile, moreover, is somewhat larger than that of the free jet, the coefficient of contraction being correspondingly higher—i.e., $C_c = (0.79)^2 = 0.63$. Since the discharge coefficients of the two jets have already been seen to be essentially the same, one is forced to conclude that the rate of flow through the orifice is essentially independent of the tangential forces exerted upon the jet by the fluid body into which it emerges. In other words, the action of the turbulent shear in causing the submerged jet to expand has no appreciable effect upon the pressure field that controls the rate of efflux. This is also evident from the experimental indication in Fig. 13 that the velocity (and hence the pressure) in the irrotational core of the jet remains unaffected by the shear in the mixing zone.

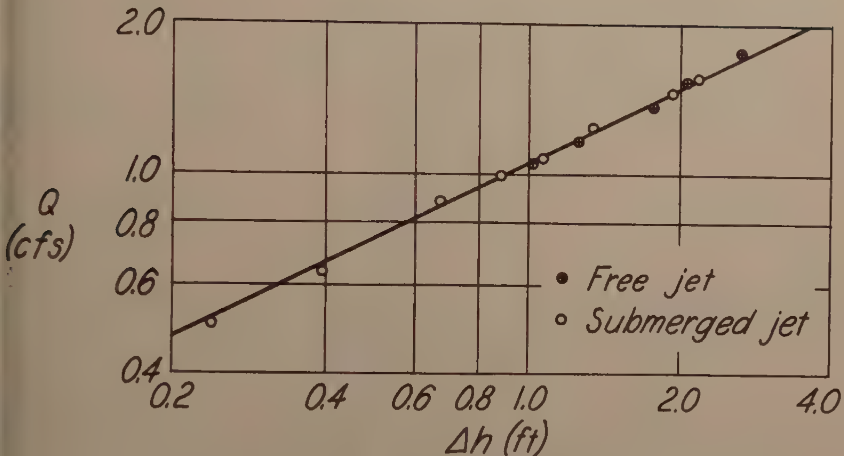

Fig. 11. Results of calibration runs

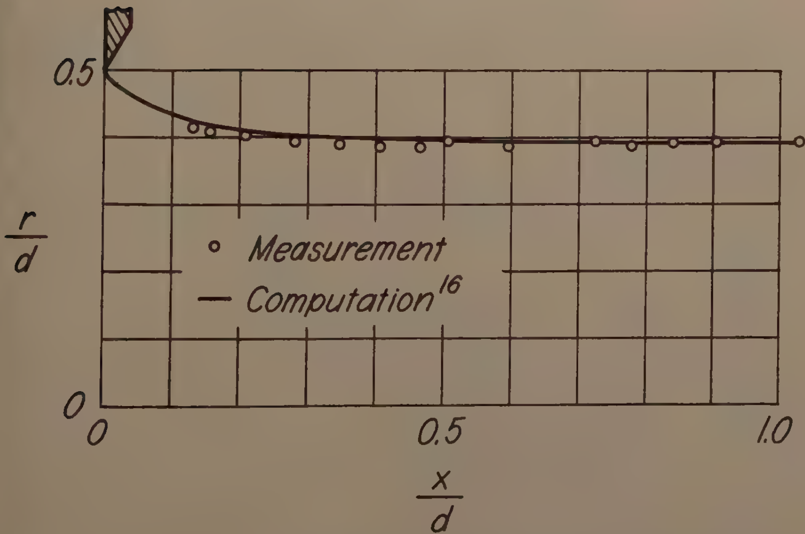

Fig. 12. Comparison of measured and computed profiles of free jet

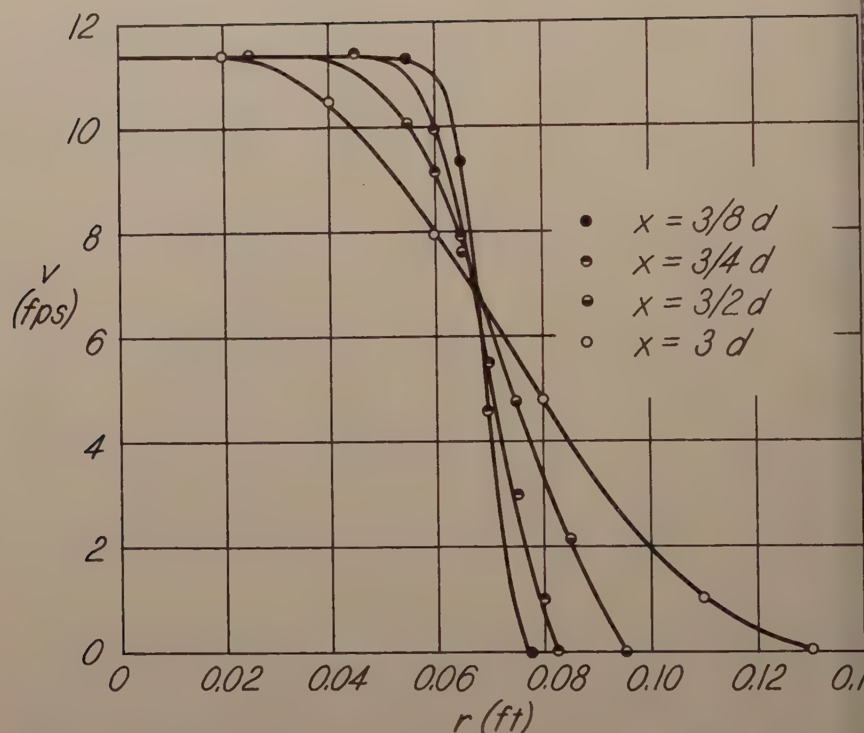

Fig. 13. Variation of velocity distribution with distanc
along axis of submerged jet

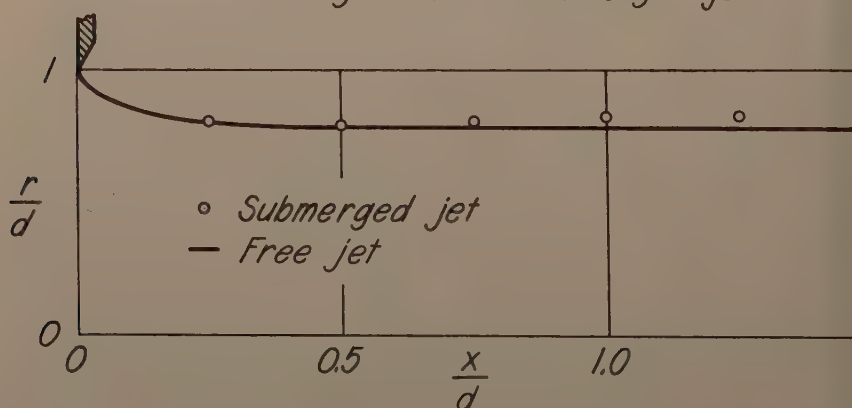

Fig. 14. Comparison of profiles of free
and submerged jets

MAXIMUM AMPLITUDE OF OSCILLATION OF A CYLINDER DUE TO ITS VORTEX WAKE

B. Guyton,[17] A. M. ASCE, B. W. LeTourneau,[18] and R. L. Player, Jr.[19]
(L. Landweber,[20] Adviser)

INTRODUCTION

When a circular cylinder is situated with its axis normal to a stream, eddies are shed periodically from alternate sides of the cylinder, generating a o-called Kármán vortex trail.[21] This gives rise to a fluctuating velocity eld and hence also a fluctuating pressure field which exerts a periodic ateral force on the cylinder.

The frequency of generation of the eddies by a non-vibrating cylinder has een determined experimentally.[22] It was found, however, that, when the ylinder is allowed to vibrate, the eddy frequency and other properties of the ake are strongly affected by the oscillation;[23] i.e., the oscillation modifies e force which causes it.

An approximate analysis of the vortex-induced vibration of a cylinder, takg into account the interaction between the vibration and the wake, was given y Steinman,[24] who devised an ingenious graphical method for displaying the odification of the vibration frequency due to the vibration. The section of s paper entitled "Effect of Vibrations on Wake and on Eddy Frequency" conudes with the statement

> "In these vortex-induced oscillations, there is no 'catastrophic range' of increasing amplification with unlimited increase of stream velocity. The greatest amplitude attainable is little more than the diameter, and may occur only in a limited critical range just above the reasonance velocity."

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9. Department of Civil Engineering, Duke University, Durham, North Carolina.
0. Research Engineer, Iowa Institute of Hydraulic Research, State University of Iowa, Iowa City, Iowa.
1. Kármán, Th. von, and Rubach, H., "Ueber den Mechanismus des Flüssigkeits- und Luftwiderstandes, Physikalische Zeitschrift, January, 1912.
2. Relf, E. F., and Simmons, L.F.G., "The frequency of the eddies generated by the motion of circular cylinders through a fluid," British Advisory Committee for Aeronautics, R. and M. 917, 1924.
3. Landweber, L., "Flow about a pair of adjacent, parallel cylinders normal to a stream," David Taylor Model Basin Report 485, July, 1942.
4. Steinman, D. B., "Problems of Aerodynamic and Hydrodynamic Stability," Proceedings of the Third Hydraulics Conference, State University of Iowa Studies in Engineering, Bulletin 31, 1946.

Steinman defines amplitude as the maximum displacement of the center of the cylinder from its undisturbed position. However, he furnishes no evidence for his statement concerning the greatest attainable amplitude.

The purpose of the present study is to enquire further, by both analytical and experimental means, into the question of the maximum amplitude of a vortex-induced vibration.

Properties of the Kármán Vortex Trail

In the range of Reynolds numbers in which laminar separation occurs on the cylinder, it is found experimentally[23] that the frequency n of vortex generation by a vibrating cylinder is given by

$$\frac{nD}{V} = \frac{0.185}{\lambda} \tag{1}$$

where D is the diameter of the cylinder, V is the free-stream velocity, and λ is the ratio b/b_0 of the width of the vortex street with and without vibration. If a is the amplitude of the oscillation, we have the experimenta ly obtained relation

$$\lambda = 1 + 1.54\,\frac{a}{d} \tag{2}$$

The force per unit length F acting on the cylinder is an oscillating quan tity whose amplitude is proportional to $\rho V^2 h$, where ρ is the density of the fluid; or, since $b = \lambda b_0$ and since, for the case of laminar separation, is found experimentally that $b_0 = 1.3\ D$, it follows that

$$F = \beta \lambda \rho V^2 D \tag{3}$$

where β is a constant of the order of unity.

Analysis of Cylinder Vibration

The vibration of the cylinder will now be analyzed on the assumption tha it is a linear harmonic oscillator satisfying the equation

$$m \frac{d^2y}{dt^2} + c\,\frac{dy}{dt} + ky = F \cos 2\pi nt \tag{4}$$

where m is the apparent mass per unit length of the cylinder (actual mass plus virtual mass), c is a damping coefficient, and k a stiffness coefficie When the vibration is steady, at frequencies near resonance, there results for the maximum amplitude

$$a_M = \frac{F}{2\pi nc} \tag{5}$$

or, from Eqs. (1) and (3),

$$a_M = 0.86 \frac{\lambda_M^2 \beta \rho V D^2}{c} \tag{6}$$

But, from Eq. (2),

$$a_M = 0.65\, D(\lambda_M - 1) \tag{7}$$

Hence, upon equating the two expressions for a_M in Eqs. (6) and (7), one obtains a quadratic equation in λ_M whose solution is

$$\frac{1}{\lambda_M} = \frac{1}{2}\left(1 + \sqrt{1 - 5.3\frac{\beta \rho V D}{c}}\right) \tag{8}$$

The largest permissible value of $5.3\beta\rho VD/c$ is seen to be unity, corresponding to which $\lambda_M = 2$ and, from Eq. (7), $a_M = 0.65$ D. This indicates that the maximum attainable amplitude is about 2/3 of the diameter.

Experiments

An attempt was made to support experimentally either Steinman's statement or this simplified theory. Since the theory as developed is based on the two-dimensional case of a rod vibrating without deformation, experimental confirmation should have been sought with a stiff cylinder supported by springs or elastic cantilever beams which would vibrate at right angles to the stream without deformation. However, time did not permit the design and construction of such an apparatus, and experiments were conducted instead on the vibration of a string. Since the string is flexible and deforms in a combination of characteristic harmonic shapes when vibrating transversely, and in addition is bowed in the direction of flow, the analysis presented above is no longer truly applicable, although it may serve as a first approximation. No analysis for the case of the string was attempted.

For the assumed condition that the foregoing theory was applicable to the string, resonance would occur when the natural frequency of the string was equal to the frequency of generation of the vortices—i.e.,

$$0.185 \frac{V}{\lambda D} = \frac{n}{2s}\sqrt{\frac{T}{m}} \quad , \qquad n = 1, 2, 3, \ldots\ldots$$

where T is the tension in the string and s its length.

Attempts were made to secure resonance using both 1/16-inch piano wire and 0.054-inch nylon string in a small air tunnel, but without success. This may have been due to the fact that the lowest fan speed available resulted in an air velocity in excess of 20 feet per second. Resonance was achieved, however, by stretching the same nylon string across the 14-inch diameter test section of the Institute water tunnel. The velocity was varied from 0 to 15 feet per second, and the vibration of the string was clearly observed through a glass window. The fundamental mode and the first and second harmonic shapes, respectively, were clearly distinguishable as the velocity was increased.

The velocity was measured with a mercury manometer reading differential pressure across the bell entrance to the test section. The amplitude of vibration, relative to the string diameter was determined by measuring with a surveyor's transit the distance between the upper and lower extreme loci of the string as it vibrated. The transit was located on the same level in order to eliminate extraneous drag vibrations, and the tripod was rubber-insulated from the vibrations of the floor. The frequency was measured by observing the vibrations with a Strobotac and recording the highest synchronized frequency. The tension of the string was measured with the water at rest by pulling normally to the string center with a spring balance, applying a known force, measuring the resultant sag, and calculating the stress by the usual methods of elementary statics. It was found difficult to apply a large tension to the nylon, because the tied end connections had a tendency to slip.

RESULTS AND CONCLUSIONS

The experimental results are presented graphically in Fig. 15, in which the dimensionless amplitude a/D and the frequency parameter or the Strouhal number nD/V are plotted against the Reynolds number VD/ν. The Strouhal numbers observed were of reasonable but somewhat high magnitude and decreased slightly with the Reynolds number, as would be predicted by the relation $nD/V = 0.185$ and the amplitude data. The amplitude data showed considerable scatter, some of which was due to the rudimentary instrumentation and some to the inherent nature of the experiment. The amplitude rose rapidly from zero at very low speed to a range of a/D from 0.25 to 0.55 at the fundamental resonance Reynolds number of 3000 predicted by using a calculated fundamental natural frequency of the string of 212 cycles per second corresponding to the 8.4-pound tension. The maximum observed dimensionless amplitude was 0.58 at velocities near fundamental resonance. A mean curve is plotted through the data. The results tend to support the foregoing simplified theory, since the predicted and observed maximum relative amplitudes are quite close: 0.65 and 0.58, respectively.

The visually observed modes of vibration are also indicated on the graph. It is to be noted that, once the maximum amplitude was attained, it remained relatively constant and independent of the mode of vibration as the velocity was increased. This indicates a sort of sustaining resonance, as was graphically explained by Steinman.[24] The transition between vibratory modes was not smooth, but there were ranges of velocity when the vibration was observed to "jump" from one mode to another.

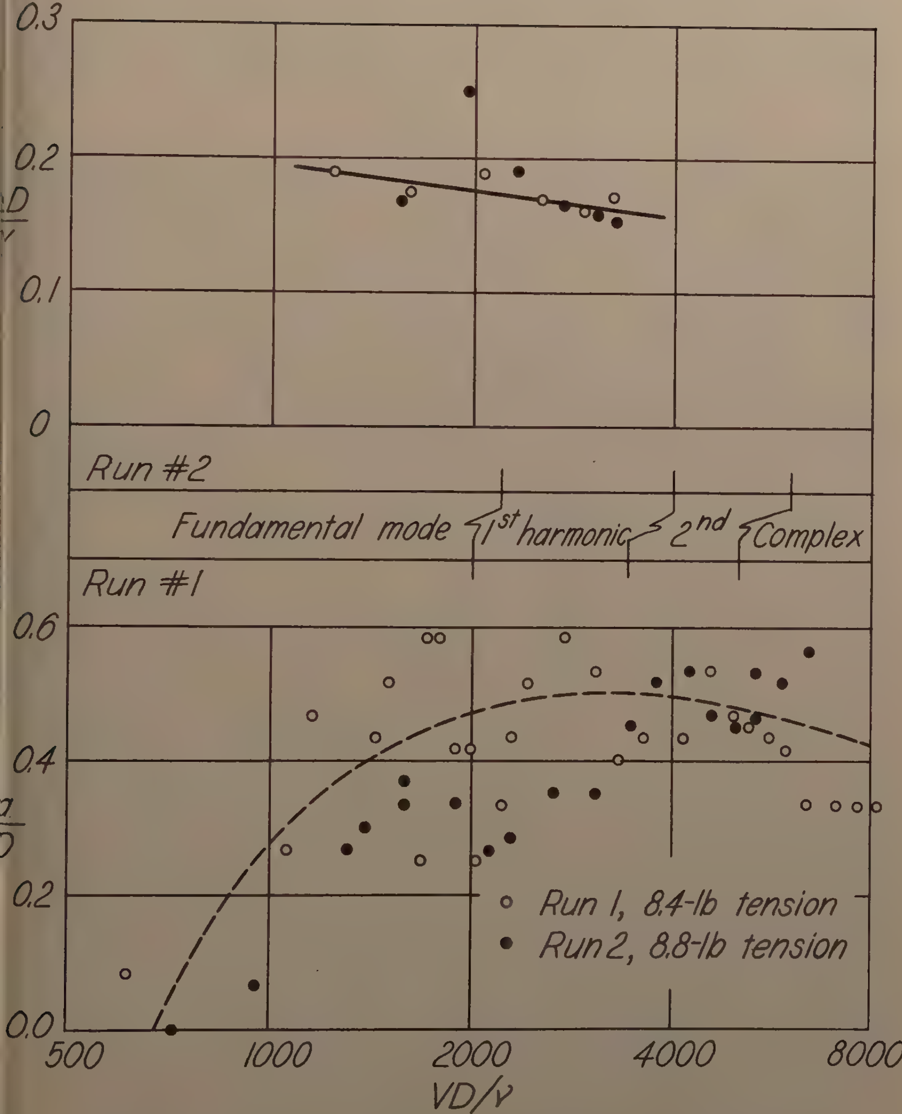

Fig. 15. Plot of Strouhal number and relative amplitude against Reynolds number

DEFLECTION OF A JET BY A NORMAL WALL

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(P. G. Hubbard,[28] J. M. ASCE, Adviser)

Although the deflection of a submerged jet by a normal wall is closely related to practical problems such as the scouring tendency of a plunging nappe, it is of interest primarily as an example of free turbulent diffusion and momentum transfer. The investigation of this phenomenon had been preceded at the Iowa Institute by a rather complete study of the flow pattern which is created by discharging a jet of fluid from an opening in a plane boundary into a semi-infinite reservoir of the same fluid.[29] In the present case, the object was to determine the streamline pattern which results when the flow is deflected by a wall normal to the initial direction of the jet.

The experimental equipment for the study consisted of two carefully rounded nozzles 0.025 and 0.05 foot in diameter which could be mounted, interchangeably, flush with the surface of an octagonal table 30 inches across. Air was supplied to the 6-inch pipe preceding the nozzle by a centrifugal fan, and the discharge was controlled by a variable-area intake to the blower. A masonite board mounted 2.5 feet above the table provided the normal wall for deflection of the jet. Velocities throughout the region of interest were measured by means of a hot-wire anemometer mounted on a mechanism for making longitudinal and lateral traverses. The pressure distribution on the normal wall was measured by an alcohol-kerosene differential manometer which could be connected to a series of piezometers arranged radially outward from the stagnation point.

Because the flow system was axially symmetrical, it could be described completely in terms of the velocity $v(x,r)$ or streamlines $\psi(x,r)$ in a meridian plane, x being measured from the plane of the issuing jet and r radially outward from its axis. From the available results of the earlier jet studies already mentioned, it was known that the determinants of the flow pattern would be the nozzle diameter D, the momentum flux of the jet $M = Q\rho v_j$, the fluid density ρ, and the distance a between the bounding planes. In order to reduce the scope of the study to correspond to the time available, a was made so large compared to the jet diameter that the jet could be considered as a point source of momentum.

A dimensional analysis of the problem, beginning with the variables

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29. Albertson, M. L., Dai, Y. B., Jensen, R. A., and Rouse, Hunter, "Diffusion of Submerged Jets," Trans. ASCE, Vol. 115, 1950.

already defined, can be outlined as follows:

$$v = f(x, r, a, \rho, M)$$

or, in dimensionless terms,

$$\frac{v}{\sqrt{M/\rho a^2}} = \varphi_1\left(\frac{x}{a}, \frac{r}{a}\right)$$

An equally useful and more easily visualized method of presenting the results is to plot streamlines in an axial plane, using the definition

$$\psi = \int_0^r v_x \, r \, dr = Q_r/2\pi$$

where v_x is the axial component of velocity and Q_r is the discharge through a circle of radius r. A pattern of streamlines can be obtained from this relationship by evaluating graphically the radii corresponding to equal increments of ψ and then plotting curves of constant ψ or of the dimensionless value

$$\frac{\psi}{a\sqrt{M/\rho}} = \frac{1}{2}\int_0^{\left(\frac{r}{a}\right)^2} \frac{v_x}{\sqrt{M/\rho a^2}} \, d\left(\frac{r}{a}\right)^2 = \varphi_2\left(\frac{x}{a}, \frac{r}{a}\right)$$

Because the hot-wire anemometer used in measuring mean velocities was oriented so that its axis was normal to an axial plane, it indicated the resultant velocity v rather than the axial component v_x required in the computations for the streamlines. This meant that two or more successive steps had to be taken in determining the location of the streamlines in the region where they were not parallel to the axis of the jet. The first approximation was obtained by assuming the measured velocity to be axial, integrating graphically, and plotting a streamline pattern on this basis. With θ' as the angle between the axis of the jet and the tangent to one of these streamlines, then a second graphical integration could be made using $v' = v \cos \theta'$. This process converged rapidly to the correct pattern, and even the second approximation was unnecessary except where the streamline curvature was pronounced.

Near the wall, of course, the streamlines curved rapidly and became practically radial where $r/(a-x) > 3$. For this region a modified method of locating streamlines was used. If v_r represents the radial component of velocity and Q_{a-x} the rate of flow through a cylindrical surface of radius r and length $a - x$, then the stream function can also be defined as

$$\psi = \frac{Q_{a-x}}{2\pi} = r\int_a^{a-x} v_r(-dx)$$

This takes the dimensionless form

$$\frac{\psi}{a\sqrt{M/\rho}} = \frac{r}{a}\int_{1-\frac{x}{a}}^{a} \frac{V_r}{\sqrt{M/\rho\, a^2}}\, d\frac{x}{a}$$

Where correction of the values for inclination of the streamlines is necessary, the process is similar to the one already outlined, except that the quantity $v' = v \sin \theta'$ is substituted in the numerator. In the region $x/a > 0.8$, $r/a < 0.3$, both methods were used as independent checks upon the accuracy of integration.

The final result for this study is shown in Fig. 16. Identical curves were obtained using each of the two available nozzles, so that the assumption of a point source of momentum was justified. In the latter circumstance the stream tube enclosing the discharge from the nozzle becomes so small that it cannot be shown on such a plot, the flow pattern hence seeming to consist almost entirely of streamlines for the fluid entrained by turbulent diffusion. The dashed line indicating the nominal boundary of the jet has the slope 1:5, which applies to a jet without the normal wall, and which also defines the limits in this case quite well.

As is evident from the diagram, the effect of the wall is felt primarily in the region $x/a > 0.8$. In spite of the concentration of the flow about the axis, the measured form of the streamlines in this region is almost identical with that computed on the assumption of irrotational flow. For the latter, the streamline equation is[30]

$$\psi = m r^2 (a - x)$$

wherein the coefficient m for the present experiments is approximately $52\sqrt{M/\rho}/a^2$.

Dimensionless values of the pressure measured on the normal wall are also plotted in Fig. 16, together with that obtained from the foregoing relationship for irrotational flow with a stagnation value of $27\ M/a^2$. Integration of this pressure distribution yielded the unexpected result that the total force on the wall is approximately 4/3 as great as the initial momentum flux of the jet rather than exactly equal to it. This apparent dynamic unbalance was probably due to the pressure reduction on the lower boundary accompanying the radial acceleration of the air drawn in by the jet. Because this negative pressure occurred over a relatively large area, its local intensity was so small that it could not be measured reliably.

On the basis of the experimental results of this study, two important conclusions can be drawn concerning the deflection of a jet by a normal wall. First, the wall has a negligible effect on the flow in an initial zone having a radius of at least one-fifth and a length of roughly four-fifths the boundary spacing. Second, in the region of pronounced streamline curvature near the stagnation point on the wall, the flow pattern may be assumed to agree with that of irrotational motion over a radial distance of about $a/8$.

30. Forchheimer, P., Hydraulik, Leipzig and Berlin, 3rd Ed., 1930, pp. 28 and 29.

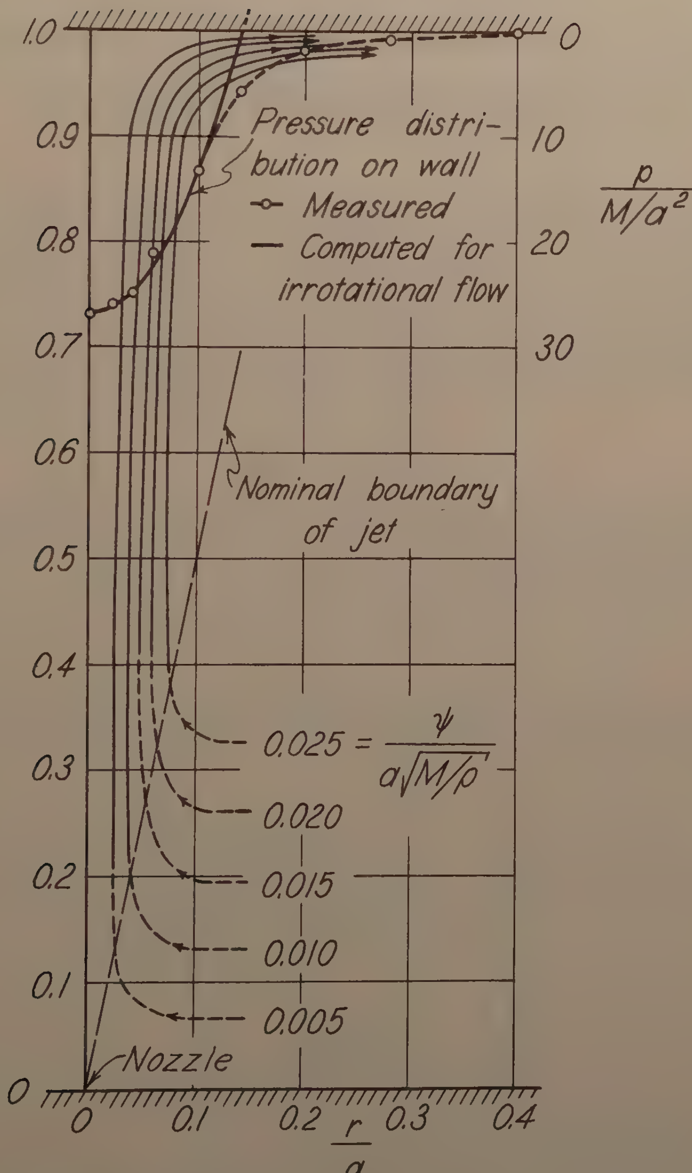

Fig. 16. Generalized characteristics of flow pattern of the deflected jet

INFLUENCE OF SLOPE AND ROUGHNESS ON THE FREE OVERFALL

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(E. M. Laursen,[12] A. M. ASCE, Adviser)

This study is concerned with the depth of flow which occurs at the end of a long channel discharging freely over a fall under various conditions of channel slope and bed roughness. The case of a horizontal channel ending in a free overfall was investigated in detail by Rouse (Civil Engineering, April 1936). The effect of channel slope was studied by Fathy and Amin (A.S.C.E. Proceedings, Separate No. 564, December 1954). A listing of a number of publications on various aspects of the free-overfall problem appeared in the February 1955 issue of Civil Engineering.

Momentum Analysis

By examining flow conditions, assumed to be two-dimensional, between the point of overfall and an upstream point x at which the pressure distribution may be taken as hydrostatic (see Fig. 17), one can express the momentum equation for a direction parallel to the bed in the following form:

$$P_f + M_f = P_x + M_x + W_L \sin\theta - F_L \tag{1}$$

If θ is taken as positive downward in the direction of flow, the above equation will apply to both downward and adverse slopes. Upon evaluating each term in turn, one obtains

$$P_f = \int_0^{y_f} p\, dy = K_1 \gamma \frac{y_f^2}{2} \cos\theta$$

where $$K_1 = \int_0^{y_f} p\, dy / (\gamma y_f^2 \cos\theta/2);$$

$$M_f = \rho \int_0^{y_f} V^2\, dy = K_2 \rho V_f^2 y_f$$

where $$K_2 = \int_0^{y_f} V^2\, dy / (V_f^2 y_f);$$

$$P_x = \gamma \frac{y_x^2}{2} \cos\theta$$

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since pressure is hydrostatic;

$$M_x = \rho \int_0^{y_x} V^2 dy = K_3 \rho V_x^2 y_x$$

where $K_3 = \int_0^{y_x} V^2 dy/(V_x^2 y_x)$;

$$W_L \sin\theta = \gamma \sin\theta \int_0^L y\,dx = K_4 \sin\theta\, \gamma y_x L$$

where $K_4 = \int_0^L y\,dx/(y_x L)$; and, finally,

$$F_L = \int_0^L \tau\,dx = \gamma \int_0^L y\,S\,dx = K_5 \gamma y_x S_x L$$

where S_f is the resistance slope at depth y, S_x is that at point x, and $K_5 = \int_0^L y S_f\,dx/(y_x S_x L)$. Equation (1) now becomes

$$\begin{aligned} K_1 \gamma \frac{y_f^2}{2} \cos\theta + K_2 \rho V_f^2 y_f &= \\ &= \gamma \frac{y_x^2}{2} \cos\theta + K_3 \rho V_x^2 y_x + K_4 \gamma y_x L \sin\theta - K_5 \gamma y_x S_x L \end{aligned} \quad (2)$$

Division of each term by γ and elimination of velocities by the continuity equation $V_f y_f = V_x y_x = q = \sqrt{g y_c^3 \cos\theta}$ then yields

$$K_1 \frac{y_f^2}{2} \cos\theta + K_2 \frac{y_c^3}{y_f} \cos\theta = \frac{y_x^2}{2} \cos\theta + K_3 \frac{y_c^3}{y_x} \cos\theta + y_x L (K_4 \sin\theta - K_5 S_x) \quad (3)$$

This equation is quite general, being identical with Eq. (1).

If the slope does not exceed 1:10, $\cos\theta$ can be taken as unity and $\sin\theta$ written as S_0. Furthermore, if the flow is turbulent, K_3 will be slightly greater than 1.0 and K_2 will be between 1.0 and K_3, because the flow is convergent. Since these terms are on opposite sides of the equation, one can take $K_2 = K_3 = 1.0$ without appreciable error. These two changes give

$$K_1 \frac{y_f^2}{2} + \frac{y_c^3}{y_f} = \frac{y_x^2}{2} + \frac{y_c^3}{y_x} + y_x L (K_4 S_0 - K_5 S_x) \quad (4)$$

If S_0 and S_x are not too large, it is reasonable to assume (subject to experimental verification) that the last term in Eq. (4) can be neglected to give

$$K_1 \frac{y_f^2}{2} + \frac{y_c^3}{y_f} = \frac{y_x^2}{2} + \frac{y_c^3}{y_x} \quad (5)$$

This is applicable to turbulent flow on both positive and adverse slopes under various roughness conditions. For steep slopes of sufficient length the depth

of the approaching flow is equal to the normal depth, so that

$$K_1 \frac{y_f^2}{2} + \frac{y_c^3}{y_f} = \frac{y_o^2}{2} + \frac{y_c^3}{y_o}$$

which can be written as

$$\frac{y_f/y_c}{2+K_1(y_f/y_c)^3} = \frac{y_o/y_c}{2+(y_o/y_c)^3} = \frac{(S_o/S_c)^{2/3}}{2(S_o/S_c)+1} \qquad (6)$$

or

$$\frac{[1-K_1(y_f/y_o)^2]\, y_f/y_o}{1 - y_f/y_o} = 2\left(\frac{y_c}{y_o}\right)^3 = 2\frac{S_o}{S_c} = 2F_0^2 \qquad (7)$$

For mild or adverse slopes y_x can be taken as equal to y_c, so that Eq. (5) becomes

$$3\frac{y_f}{y_c} = 2+K_1\left(\frac{y_f}{y_c}\right)^3 \qquad (8)$$

Experimental Equipment and Procedure

The experimental investigation was carried out in a glass-walled tilting flume 30 feet long and 2 feet wide. Discharges were measured by means of an orifice meter and were selected to give simple numerical values of the critical depth for parallel flow. In the preliminary tests seven values of the critical depth ranging from 0.1 to 0.4 foot were used, while the final tests were confined to discharges corresponding to 0.2 and 0.4 foot. Ten slopes in the range +0.03 to -0.02 were tried. Two roughness conditions were investigated: (a) the relatively smooth stainless-steel bottom of the flume, and (b) an artificial roughness consisting of 1/4x1/4-inch brass bars fixed transversely along the bottom of the flume. Previous tests by Walter Rand of the Institute staff had indicated that with this artificial roughness in position the virtual bottom of the channel was at the mid-height of the roughness bars. Accordingly, the crest of the overfall was arranged as in Fig. 18, so that the crest level would be in line with the virtual bottom of the approach channel.

The standard experimental procedure was as follows. The discharge was set to give the required value of the critical depth for parallel flow and kept constant while the slope was varied throughout its range. For the horizontal case and for the maximum positive and negative slopes, enough point-gage readings were taken to determine the complete surface profile. For the intermediate slopes, enough readings were taken to determine y_f , L , and the slope of the water surface at the overfall crest and at the point x.

The scatter in the results obtained seemed to be within the limits to be expected for the experimental set-up involved. The fluctuations in the air-water manometer used in measuring the discharge could easily result in

an error of 2% or more in the discharge and hence a corresponding error in the value of the critical depth and critical slope. For flows near the critical and for the steeper slopes the water surface was quite disturbed and the depth as determined by point-gage readings could be in error by 1 or 2%.

DISCUSSION OF RESULTS

The experimental results can be first used to justify the dropping of the last term in Eq. (4) - namely, $L\,y_x(K_4S_0-K_5S_x)$. From the detailed profiles it was found that, as y_f increased towards y_x, K_4 varied from 0.90 to 0.98 and K_5 varied from 1.25 to 1.05. It was noted that the following approximations based on a parabolic profile gave good results:

$$K_4 = \frac{\int_0^L y\,dx}{y_x L} = \frac{2 + y_f/y_x}{3}$$

$$K_5 = \frac{\int_0^L dx/y^2}{L/y_x^2} = \frac{2 + (y_x/y_f)^2}{3}$$

For steep slopes the term under examination is $L\,y_x\,S_0\,(K_4 - K_5)$, which varies from $-0.015\,y_x^2$ to $-0.005\,y_x^2$. For mild slopes its value varies from $-0.02\,y_x^2$ to $-0.04\,y_x^2$ and in sixteen of the twenty cases measured lies between $-0.025\,y_x^2$ and $-0.035\,y_x^2$. The term is clearly of the same order (and opposite sign) as the approximation involved in taking $K_3 = 1$ and may be neglected in a first approximation to the solution.

The critical slope S_c for two-dimensional flow is given by

$$S_c = \frac{f}{8} = \frac{g}{C^2}$$

In evaluating the critical slope in the present tests, the value for the resistance coefficient f for the rough case was taken from an equation derived by Walter Rand for the same artificial roughness placed in the same flume, while for the smooth case C was taken from a formula proposed by R. W. Powell. The two formulas are

Rough: $$\frac{1}{\sqrt{f}} = 2.04 \log \frac{y_o}{k} + 1.35$$

Smooth: $$C = 42 \log \frac{R}{C}$$

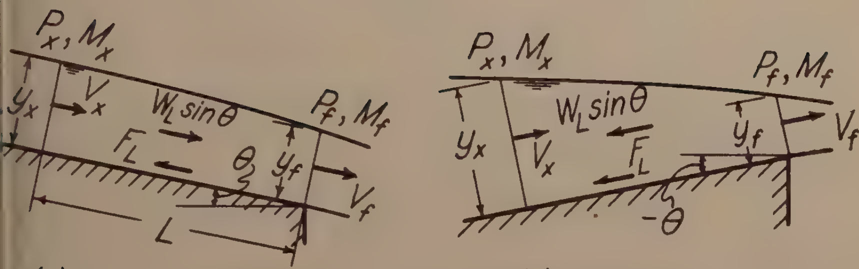
The values of y_f measured in the final series of tests are plotted in Fig. 19 in terms of y_f/y_c and S_0/S_c. It will be seen that the points representing widely varying conditions of roughness and slope define a single line. This is to be expected in the case of steep slopes, as the plotting is in terms of the factors occurring in Eq. (6). The values for mild and adverse slopes also plot fairly well about a line, but the small differences involved make it

difficult to determine whether the scatter which occurs is systematic or not. In the case of adverse slopes, less scatter is obtained if y_f/y_c is plotted against $S_0 + S_c$ (which will be negative when $S_0 < - S_c$), but further experimentation would be required to settle this question. Accordingly, the results for steep, mild, and adverse slopes are all plotted in Fig. 19 in terms of y_f/y_c and S_0/S_c.

The corresponding relationship based on Eqs. (6) and (8) cannot be determined unless values for the pressure variable K_1 are known or assumed. The experiments indicated that K_1 decreased in value as the slope passed from adverse to horizontal and from horizontal to mild and then to steep. A good correspondence with the experimental points (note the plotted curve) is obtained if the following values of K_1 are assumed:

$$S_0/S_c < -5 \qquad K_1 = 0.60$$
$$-5 < S_0/S_c < 1 \qquad K_1 = 0.30 + \sqrt{1 - S_0/S_c}\,/8$$
$$1 < S_0/S_c \qquad K_1 = 0.30$$

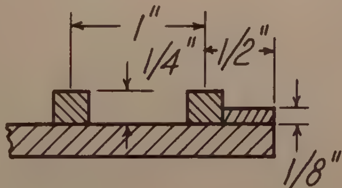
However, the validity of assuming these values must still be checked by direct pressure measurements.



(a) Positive slope (b) Adverse slope

Fig. 17. Definition sketches

Fig. 18. Details of bar roughness and overfall crest



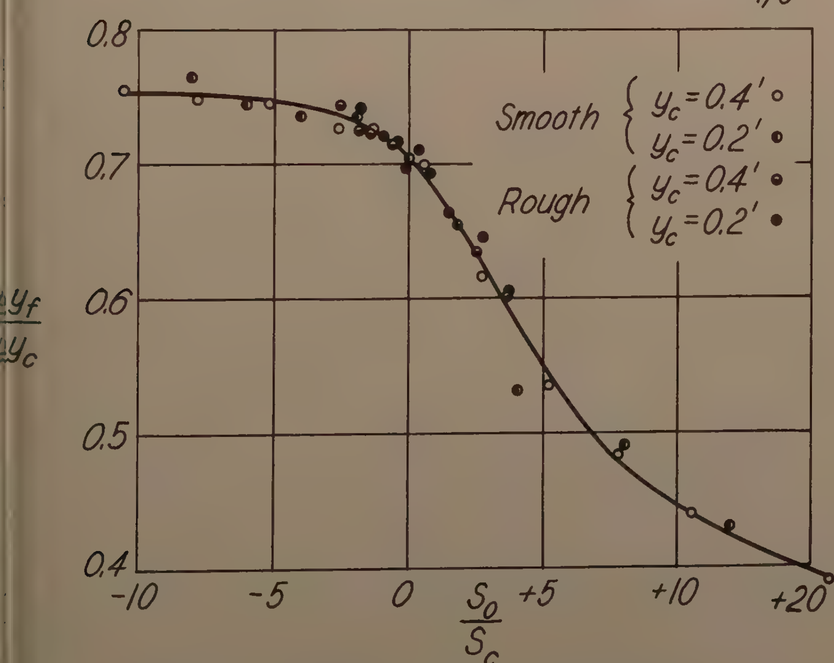


Fig. 19. Composite plot of test results

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A NEW DEVELOPMENT IN FLOW MEASUREMENT: THE DALL FLOW TUBE

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(Proc. Paper 1039)

Several new types of flow-metering devices of the pressure-difference or ierential type for pipe flow measurements have been developed in recent ars.

Of particular interest to civil engineers is the Dall Flow Tube which, as own by Fig. 1, is a modified Venturi Tube. Originally built and studied in gland[1] its characteristics were first discussed before an American audi- ce by I. O. Miner of Builders-Providence, Inc.[2]

Essential differences between the Dall Flow Tube and the classical Venturi ibe are:

1) The steepness of both the converging cone and of the diverging or pressure-recovery cone, the former having an included angle of approximately 40° and the latter an included angle of approximately 15°.
2) The presence of a dam at (a) and the characteristic location of the up- stream pressure connection, similar to the type commonly used in Europe and known as "corner taps."
3) The sharp edges at (b), (c), (d) and (e).
4) The slot between (c) and (d), through which the downstream or throat pressure connection is made.
5) The sudden enlargement at (f). By truncating the recovery cone and because of the steepness of the converging and recovery cones, a very compact instrument is obtained, the length of the whole device being of the order of two pipe diameters only.

Experimental data indicate that the differential pressure produced by a ll Flow Tube is higher than that of a conventional nozzle or Venturi Tube. e ratio of Dall Flow Tube differential to Venturi Tube differential varies om 1.91 for a diameter ratio β of 0.5[3] to 2.43 for $\beta = 0.8$.

ce: Discussion open until January 1, 1957. Paper 1039 is part of the copyrighted Journal of the Hydraulics Division of the American Society of Civil Engineers, Vol. 82, No. HY 4, August, 1956.

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By H. E. Dall, Research Engineer for George Kent, Ltd.

I. O. Miner: "The Dall Flow Tube," Trans. A.S.M.E., 78, no. 3, April 1956.

β is the ratio of throat diameter to pipe diameter: $\beta = d/D$.

An unexpected characteristic of the Dall Flow Tube is that the nonrecover-able pressure head, or head loss of the instrument is extremely low. Fig. 2 gives a comparison, established by Miner, between the head loss, expressed in per cent of the differential, for the Dall Flow Tube and two types of Venturi Tubes. In the standard Herschel-type long form Venturi Tube, the included angle of the recovery cone is 5° 5'. In the short form Venturi Tube, this angle is appreciably larger. Furthermore, the recovery cone is truncated to obtain a more compact instrument. The solid lines on Fig. 2 indicate the head loss in per cent of differential as a function of the diameter ratio β. The dotted line gives the loss through a Dall Flow Tube producing the same differential as that of a Venturi Tube with the value of β given by the abcissa.

The advisability of expressing the head loss in per cent of the differential may appear questionable because of the high value of the differential itself. Another comparison, based on the ratio of head loss in Dall Flow Tubes to head loss in Herschel type Venturi Tubes, long form for equal flow rates (at pipe Reynolds Numbers larger than 350,000) indicates that, for a given value of β, the head loss for the Dall Flow Tube is smaller when the diameter ratio is greater than 0.552. No satisfactory explanation has been given yet of the low pressure loss of the Dall Flow Tube. Several studies are in progress, trying to obtain a better understanding of the fluid mechanics principles involved in the flow through a Dall Flow Tube.

Dall Flow Tube Discharge Coefficients

The relationship between rate of discharge and differential head is expressed in the most general case of compressible fluids by the classical equation for inferential flow meters:

$$q = \frac{CYa}{\sqrt{1 - \beta^4}} \sqrt{2g.h}$$

in which

q = volumetric rate of discharge

a = throat diameter = $\frac{\pi \ d^2}{4}$

C = coefficient of discharge

Y = expansion factor

β = diameter ratio = d/D

g = acceleration of gravity

h = differential pressure head

It is well known that the coefficient of discharge C is a function of the type and proportions of the instrument and of flow characteristics which combine in the expression of the pipe Reynolds number:

$$R_D = \frac{VD}{\nu}$$

V = average velocity of flow in the pipe

D = pipe diameter

ν = coefficient of kinematic viscosity of the fluid

The expansion factor Y takes into account the compressibility of the fluid. It is a function of the diameter ratio β, of the ratio of throat to inlet pressure and of the adiabatic exponent k. For incompressible fluids, Y has the value of unity.

Values of C, as obtained by Miner from his experiments are given by the solid line on Fig. 3. Those values are for Reynolds numbers larger than 350,000.

The author has had the opportunity to calibrate several Dall Flow Tubes in the Hydraulic Laboratory at Cornell University. One of them was of large size: 24.02 in. x 17.544 in. (β = 0.7304). This tube was calibrated by measuring flows between 5.62 and 20.50 cfs volumetrically in a standpipe 6 ft. in diameter and 55 ft. high providing a tank of 1500 ft^3 useful capacity. A suitable diverter equipped with an electrical timing device was used. Differential pressure heads ranged from 0.282 to 3.80 ft. of water. The results of that calibration are given on Fig. 4.

The curve shows the characteristic trend indicated by Miner, the coefficient of discharge decreasing for increasing Reynolds number and reaching a constant value: C_c at high Reynolds numbers.

The constant value of the coefficient of discharge is $C_c = 0.661$, which agrees very well with Miner's data from Fig. 3.

Other calibrations were part of a research project on inferential meters conducted in the Hydraulic Laboratory at Cornell University with the sponsorship of Builders-Providence, Inc. The tests were all conducted with 8 in. pipe size and the flows measured volumetrically. The results of those tests are summarized in Table I giving the dimensions of the Dall Flow Tubes tested, the value of Reynolds number at which the coefficient of discharge becomes constant: $(R_D)_n$ and the constant value of the coefficient of discharge: C_c. The last three lines apply to the instrument known as the Dall Insert Nozzle which is a Dall Flow Tube inserted between flanges in the pipe line.

It should be noted that the values of $(R_D)_n$ differ somewhat from the value of 350,000 indicated by Miner. The difference, however, is small except for two of the largest diameter ratios.

A comparison of the experimental values of C_c with the data from Fig. 3 indicates a very satisfactory agreement except for the Dall Insert Nozzle with the largest diameter ratio. Part of this difference may be accounted for by the fact that inferential flow meters of large diameter ratio are more sensitive to the conditions of installation than those of small diameter ratio.

Effect of Two Short-Radius Elbows in Orthogonal Planes

Miner has investigated the effect of various upstream disturbances on the behavior of the Dall Flow Tube, namely the effects of a single elbow, of a gate valve partly opened, and of an increaser or a decreaser.

The author studied the effect of two short-radius elbows in orthogonal planes.

The test run was an experimental pipe, 8 in. in diameter, under a static head of 80 ft.

Tests were conducted both without and with straightening vanes at the

upstream end of the approach pipe, immediately after the two elbows. These straightening vanes consisted of four radial baffles approximately 16 in. (2D) long.

The short-radius elbows themselves were preceded by sufficient lengths of straight pipe to insure that they alone would affect the behavior of the instrument tested.

The results of the tests are summarized in Table II. L_a is the length of approach pipe between the two elbows and the tube tested, D the pipe diameter, C the coefficient of discharge and α the difference (in per cent) between the coefficient of discharge for a particular condition and that obtained for the same tube with a length of approach pipe 27 ft. 7/8 in. (41D) long.

It should be noted that any value of α smaller than 0.2 per cent is insignificant, being smaller than the possible experimental error.

The study indicates that the effect of two short radius elbows in orthogonal planes on the performance of Dall Flow Tubes is only slightly larger than their effect on standard Venturi tubes. The disturbance appears more significant for larger diameter ratios. The effect of straightening vanes is to raise the coefficient.

Comparison of the values of the coefficient of discharge obtained without and with straightening vanes gives the following differences:

0.15 per cent for the 8 in. x 4 in. Venturi Tube
0.47 per cent for the 8 in. x 4.4 in. Dall Flow Tube
0.31 per cent for the 8 in. x 5 in. Venturi Tube
0.90 per cent for the 8 in. x 5.2 in. Dall Flow Tube
0.73 per cent for the 8 in. x 6 in. Venturi Tube

Tests are now under way to study the effect of two elbows on a Dall Flow Tube of very large diameter ratio.

Expansion Factor Tests

A determination of the value of the expansion factor Y is essential to the use of Dall Flow Tubes with compressible fluids. Since no theoretical analysis is available to this effect, the values of the expansion factor must be obtained experimentally. This was done by the author, using air flows. The procedure was as follows:

Various Dall Flow Tubes were first calibrated in the Hydraulic Laboratory, using water. The Dall Flow Tubes were installed in series with Venturi Tubes of known characteristics. Suitable lengths of approach pipes were provided both for the Venturi Tubes and for the Dall Flow Tubes to eliminate the effect of approach conditions. After the water calibration, each installation was installed in an air compressor line. Since the expansion factors for Venturi Tubes are known[4] the Dall Flow Tube expansion factors could be determined by comparison. The results are given in Fig. 5. Values of the expansion factor Y are plotted as a function of x/k.

x = ratio of differential pressure to absolute inlet static pressure

k = adiabatic exponent

In each case, the Dall Flow Tube expansion factor may be compared with that

4. A.S.M.E. Power Test Code, Part 5, Chapter 4.

of a concentric orifice of equal diameter ratio.[5] The conclusion is that under the conditions investigated the values of the expansion factors for Dall Flow Tubes and concentric orifices are quite comparable.

CONCLUSIONS

The Dall Flow Tube, as now constructed, thus offers the two-fold advantage of a great compactness and a low head loss. It is only slightly more sensitive to the conditions of installation than standard Venturi Tubes.

For Dall Flow Tubes with throat to inlet diameter ratios β not more than 0.75 (usual limit for Venturi Tubes), the coefficient of discharge can be predicted within 1%. For larger values of β, direct calibration under upstream installation conditions is recommended, unless larger tolerances on coefficient value are acceptable.

5. A.S.M.E. Power Test Code, Part 5, Chapter 4.

Table I

Coefficients of discharge

Dall Flow Tube			$(R_D)_n$	C_c	C_c from Fig. 3 (Miner)	Diff. (%)
D (in.)	d (in.)	β	(2)	(3)		
24.02	17.544	0.7304	700,000	0.661	0.660	+ 0.2
8.00	4.397	0.5496	300,000	0.710	0.705	+ 0.7
8.00	5.217	0.6521	400,000	0.690	0.684	+ 0.8
8.00	6.336	0.7920	700,000	0.627	0.633	- 1.0
8.00(1)	4.078	0.5098	400,000	0.715	0.710	+ 0.7
8.00(1)	5.217	0.6521	400,000	0.684	0.684	0
8.00(1)	6.338	0.7923	450,000	0.620	0.633	- 2.1

(1) Dall Insert Nozzle.

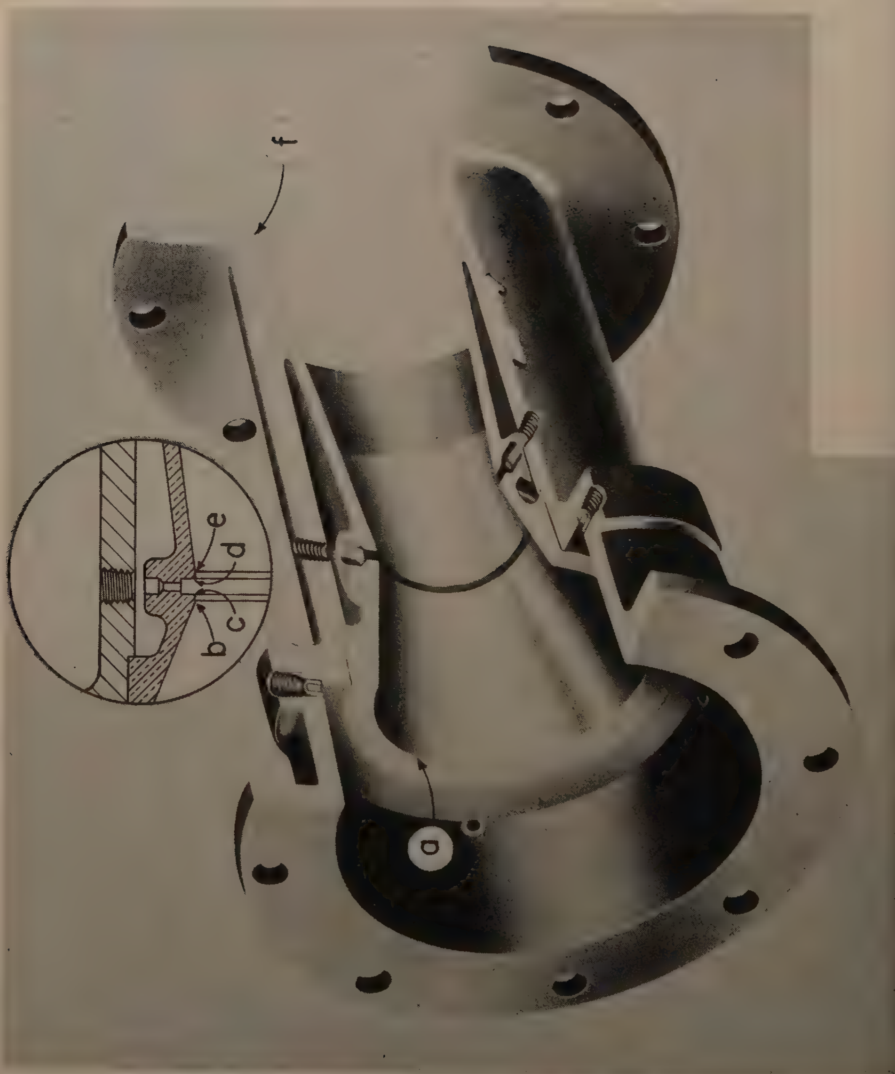
(2) Value of pipe Reynolds number at which the coefficient of discharge C becomes constant.

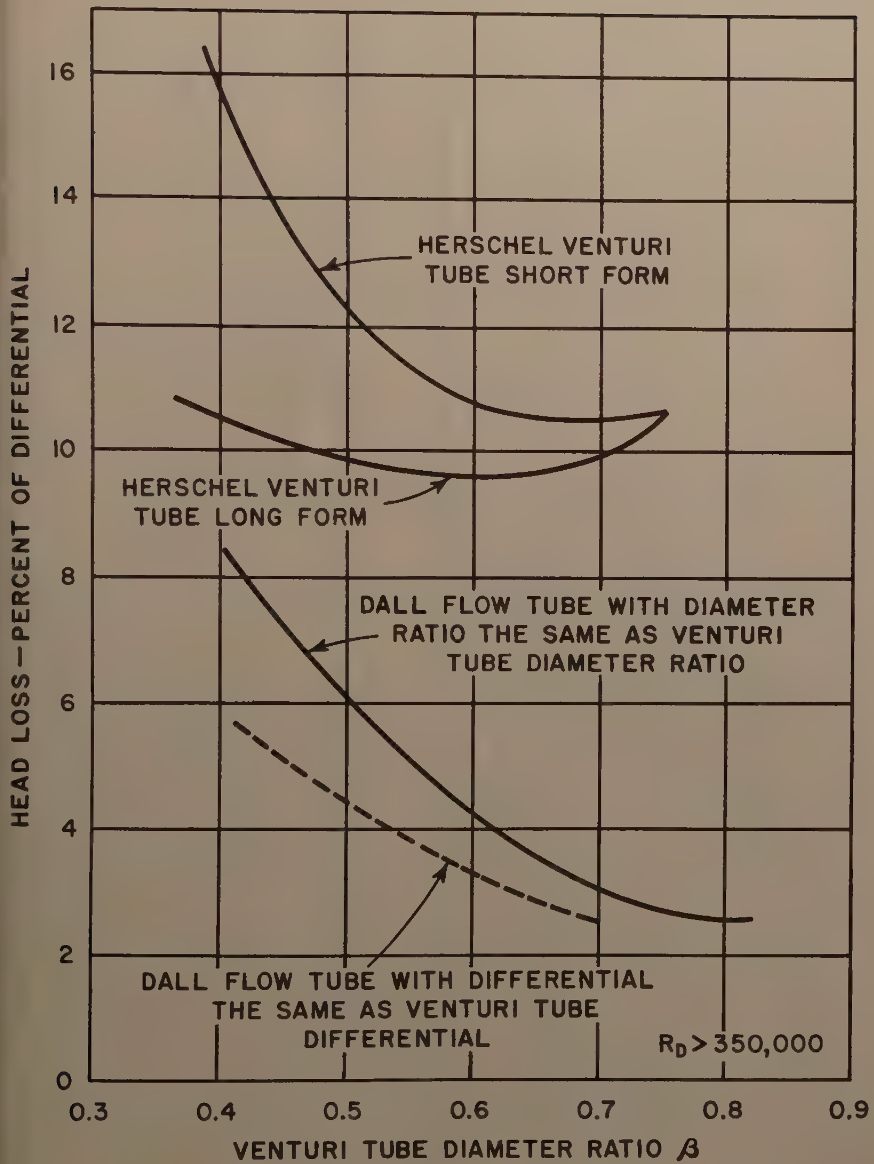
(3) Constant value of the coefficient of discharge.

Table II

Effect of two short-radius elbows in orthogonal planes

L_a ft-in	L_a/D	8 in. x 4 in. Venturi Tube		8 in. x 4.4 in. Dall Flow Tube		8 in. x 5 in. Venturi Tube		8 in. x 5.2 in. Dall Flow Tube		8 in. x 6 in. Venturi Tube	
		C	α	C	α	C	α	C	α	C	α
Tests without straightening vanes											
0	0	0.9834	-0.02	0.7117	+0.24	0.9843	-0.34	0.6862	-0.48	0.9782	+0.09
4-4	6.5	0.9833	-0.03	0.7096	-0.05	0.9836	-0.41	0.6859	-0.52	0.9728	-0.46
10- 6-3/4	15.5	0.9832	-0.04	0.7103	+0.05	0.9851	-0.26	0.6877	-0.26	0.9768	-0.05
19- 4-1/8	28	0.9841	+0.05	0.7106	+0.09	0.9860	-0.17	0.6909	+0.20	0.9780	+0.07
27- 7/8	41	0.9836	. . .	0.7100	. . .	0.9877	. . .	0.6895	. . .	0.9773	. . .
Tests with straightening vanes											
0	0	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
4-4	6.5	0.9840	-0.12	0.7090	-0.43	0.9885	-0.23	0.6881	-1.10	0.9776	-0.69
10- 6-3/4	15.5	0.9846	-0.06	0.7127	-0.09	0.9910	+0.02	0.6950	-0.10	0.9821	-0.23
19- 4-1/8	28	0.9851	-0.01	0.7136	+0.05	0.9907	-0.01	0.6969	+0.17	0.9846	+0.02
27- 7/8	41	0.9852	. . .	0.7133	. . .	0.9908	. . .	0.6957	. . .	0.9844	. . .


f
e
d
c
b
a



HEAD LOSS COMPARISON

FIG. 2

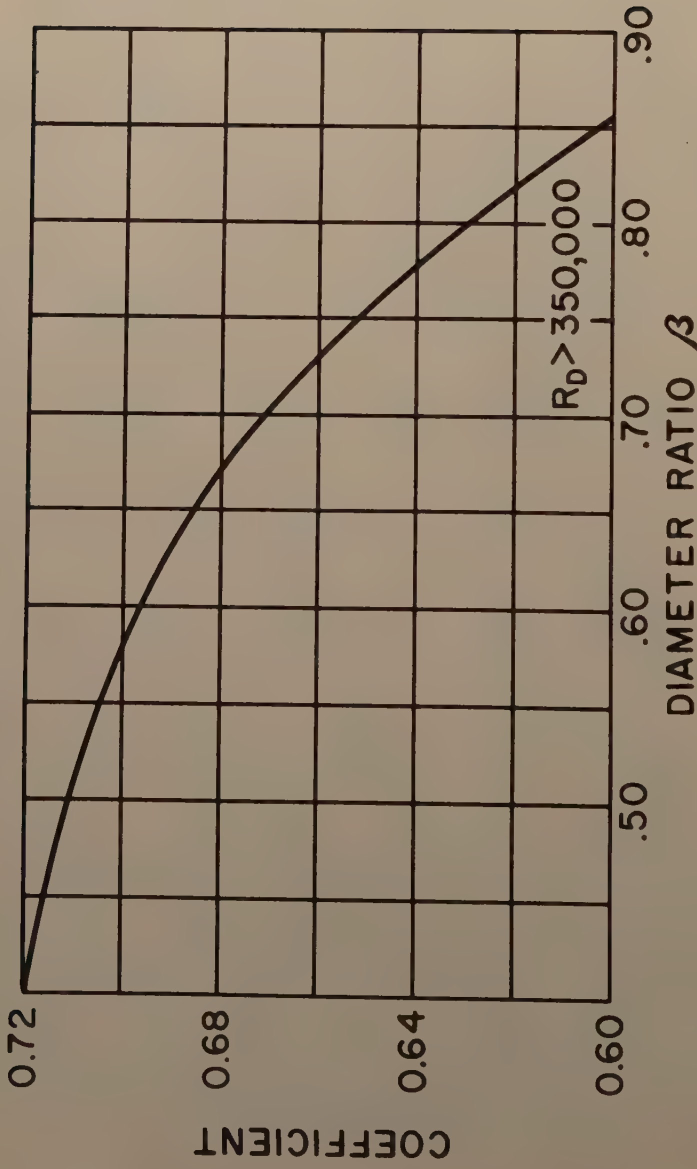
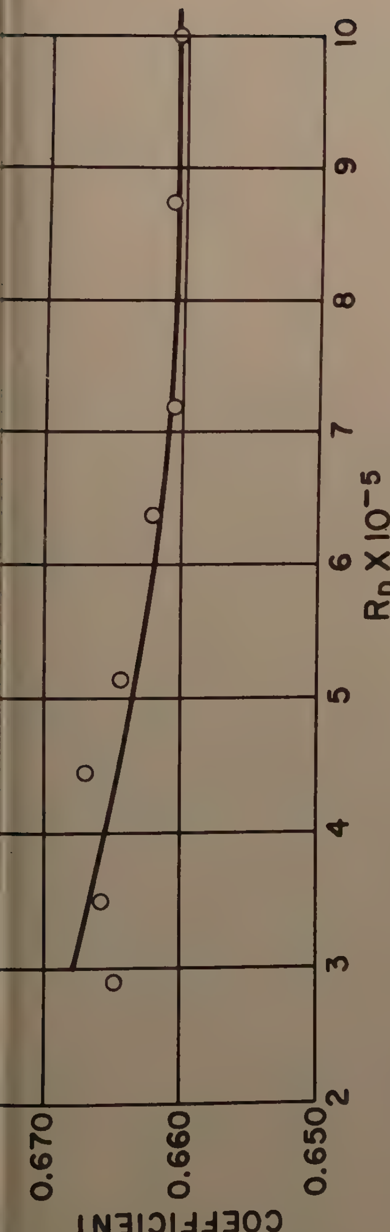

COEFFICIENT
0.72
0.68
0.64
0.60
.50
.60
.70
.80
.90
DIAMETER RATIO β
$R_D > 350{,}000$

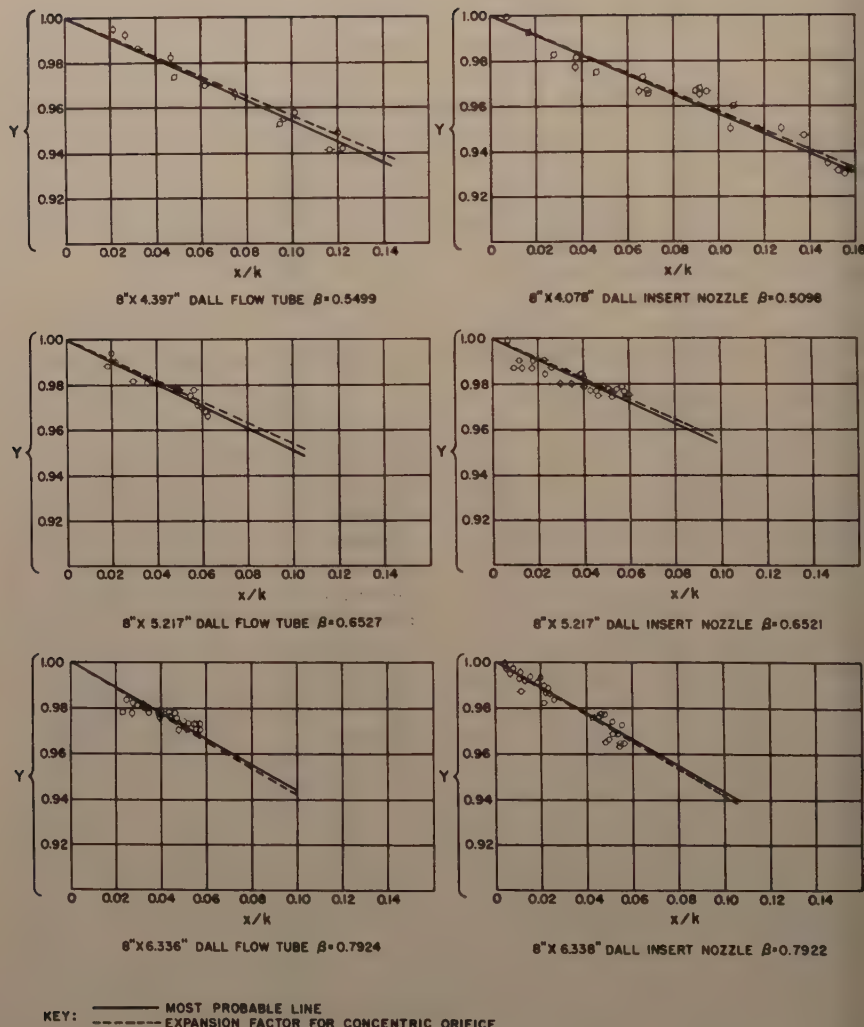
FIG. 3



CORNELL UNIVERSITY
SCHOOL OF CIVIL ENGINEERING
DEPARTMENT OF HYDRAULICS AND HYDRAULIC ENGINEERING

TEST OF
24.025 IN. X 17.544 IN.
DALL FLOW TUBE
BUILDERS - PROVIDENCE, INC.

FIG. 4



DALL FLOW TUBE EXPANSION FACTOR

FIG. 5

Journal of the
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FLOOD PLAIN ASPECTS OF RIVER PLANNING

Anthony M. Lunetta,* A.M., ASCE
(Proc. Paper 1040)

A proper evaluation of the role of the flood plain in river planning involves a consideration of all aspects of water resources. Such river problems as transportation, power, recreation, water supply, sewage disposal, pollution, flood control and sedimentation affect the welfare of the nation. A study of all the problems connected with and involving streams is beyond the scope of this paper. Instead, only one aspect of the flood control problem will be discussed, namely, the use and control of the flood plains through the medium of zoning and the problems encountered.

Regulating or prohibiting the use of the flood plain for dwellings or other property which would be seriously damaged by flooding has been proposed at various times since 1900. With the acceleration of house construction in the late 1940's, the use of zoning to control the flood plain has increased in the United States. The concept of flood plain zoning has its greatest use as a preventive measure in areas which are undeveloped and sparsely developed. The presence of buildings and other structures of various types may make it more economically feasible to provide protection to the area rather than to evacuate the area. The nature of the zoning procedure takes many forms and is administered and enforced by numerous governmental units and agencies. The range of flood plain zoning varies from control of encroachments on the stream itself to the prohibition of the use of any portion of the flood plain for any activity or structure which will adversely affect flood flows. In some cases, where zoning powers were believed to be insufficient to restrict the use of the flood plain, the areas were purchased as public lands and developed for parks and other recreational facilities.

Encroachment on the flood plains of rivers and streams has occurred since the beginning of civilization. The earliest settlements, requiring water for a multiplicity of uses, such as transportation, potable water, protection and waste disposal, grew up along the banks of rivers, often on the flood plains.(1) These settlements established economic values and social roots which have withstood, in most instances, the destructive forces of floods. But floods are periodic and persistent, and necessitate action by man to minimize or prevent

Note: Discussion open until January 1, 1957. Paper 1040 is part of the copyrighted Journal of the Hydraulics Division of the American Society of Civil Engineers, Vol. 82, No. HY 4, August, 1956.

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the damages caused by excessive storm runoff. Sometimes this action has been directed towards protecting these values and roots without determining if the protection, when all aspects are considered, is warranted. The net result is a continuous contest of man opposing the forces of nature and in many instances the results are far from satisfactory.

The expense involved, initially and subsequently, of providing the protection against floods has reached the point where the wholesale evacuation of the flood plain is being considered as the most economical solution to the man-made problem of flood damages. Some areas, benefiting from the experiences of their neighbors, are attempting to forestall future flood problems by maintaining as much of the flood plain as is economically feasible for the passage of flood waters.

The Role of the Flood Plain

The flood plain plays an important part in the development of a society. The environmental features of the flood plain lend themselves to manifold uses which have a direct bearing on the growth of cities and the development of the area. The proper uses of the flood plain varies from area to area and depends on the climate, geography and natural resources, as well as on the existing and potential development of the non-flooded area. The solution of stream problems is usually directed towards the maximum utilization of the flood plain. This may involve the inundation of agricultural lands by storage reservoirs on one part of the stream to protect highly developed industrial and residential areas situated on the flood plains of another portion of the stream. The social, psychological and economical problems arising from such a solution are exemplified in the controversy which resulted from the proposed Tuttle Creek dam on the Big Blue River in Kansas to protect the cities of Topeka and Kansas City.

Planning, in its best sense, entails full knowledge of the problem, its causes, its symptoms and it cures. So with river planning, it is necessary to understand the behavior of streams and the characteristic features of their drainage basins in order to plan effectively for the maximum utilization and protection of the water resources available. In addition, procedures must be established to minimize the dangers from floods, an excess of water within a relatively short period of time.

A stream is a constantly moving body of water providing drainage for all parts of the watershed. Storm runoff, from roofs, lawns, streets, farms and other surfaces, eventually finds its way to a natural drainage ditch or man-made storm sewer and then into a stream. Past experience has repeatedly shown that isolated sections of a stream cannot be altered without regard for adjacent reaches of the stream. A drainage basin is a closely correlated natural unit and should be studied and treated as a unit. The land along any stream may be divided into three zones:(2)

1) The normal stream channel.
2) The flood plain, which is usually dry but occasionally flooded.
3) The high land, which is never flooded.

In general, the stream channel is encroached upon less frequently than the flood plain. Many of the stream channel encroachments are bridges necessary to provide continuous and uninterrupted transportation. These bridges, if properly designed from a hydraulic viewpoint, nullify, to a considerable

xtent, any adverse effect on flood flows. Most of the remaining stream hannel encroachments occur along banks of streams. Structures, including ills, which extend into the existing stream channel frequently restrict the tream to the point where higher flood stages result. The economic and hysical obstacles involved in constructing structures adjacent to streams ave deterred a greater number of encroachments on streams. The flood lain, on the other hand, does not have these obstacles unless it is swampy. ince the flood plain is usually dry, sometimes for years, there is considerble encroachment on this area. The damage to these encroachments during imes of flood account for a major portion of the total flood damages. Thereore, if future encroachment on the flood plain is not controlled, flood damges will increase rapidly and the cost of providing protection will also increase.

In many places, encroachment in the flood plain has been encouraged by a egal doctrine which permits man to invade not only the flood plain but even ne stream channel itself, despite recurring and increasing flood losses. While some of this encroachment has been made in ignorance of the flood azard, most of it has resulted from attempting to utilize the resources of ne terrain, soil and water.(3) Farmers found easy and rich soil to till, nanufacturers found cheap land, convenient transportation and power, raiload companies found easy grades on which to lay their tracks. Following ne usual tendency, businesses and people moved into the flood plain area to e near the center of activity. Largely through inertia, since some activities which are now in the flood plain are there uneconomically, many of these activities have remained in the area.

Regulation of flood plain use has not been extensive because of:

1) Lack of sufficient flood data.
2) Lack of land use studies in flood plains.
3) Objections of flood plain property owners and users to any measures limiting land use.
4) Lack of legal precedent to prevent or restrict land use in the flood plain.

Hydrologic and Hydraulic Aspects

The engineer, administrator and planner, in considering flood plain zoning, s interested principally in the following hydrologic and hydraulic information nd data:

1) Flood seasons.
2) Duration of flooding.
3) Flood stages.
4) Flood velocities.
5) Extent of flooding.
6) Minimum and/or economical channel widths.
7) Frequency and magnitude of the greatest flood.
8) Potential greatest flood.

Knowledge of the flood seasons and probable duration of flooding will overn to a large extent what uses may be permitted on a flood plain. Flood tages, flood velocities, and the extent of flooding will assist in delineating he flood plain and in determining the ground floor elevations and the type of

buildings to be permitted in the flood plain. The extent of flooding and flood stages are generally required for the zoning plan. The last three items are required to arrive at an economical appraisal of the value of zoning a flood plain.

Nearly all floods are caused by excessive rainfall, which may be augmented by water from melting snow in some areas. Heavy rains result from a combination of meteorological factors which may occur at any time in an area. Meteorologic and climatic factors vary from area to area and determine the type of storms to be expected, the time of occurrence of such storms, and the probable severity of each type of storm in each area. The two principal types of storms which produce enough rainfall to cause floods are the thunderstorm and the stagnant-cold-front storm. Combinations or variations of these two types of storms, in addition to simple lifting of warm moist air masses over mountains, may result in flood-producing rains.

The physical characteristics of drainage basins vary considerably from one basin to another. The upper portions of many drainage basins have relatively steep gradients. In these areas floods will have a rapidly rising and falling flood hydrograph. The flood plains are narrow with little storage and velocities are high. Large quantities of debris and sediment may be transported for considerable distances. As the streams approach the sea, the gradients decrease and the flood hydrograph has a long period of rise, a broad peak, and a longer period of fall. Considerable flood plain storage is usually present; inundation of the flood plains may be protracted and sediment and debris transported from the upper reaches may be deposited on the flood plains. The rivers tend to meander and frequently change their course

It is impossible to separate the flood plain from the river itself in order to consider its hydrologic and hydraulic aspects. All streams have flood plains along their entire length, although their width may vary from zero, in reaches which have perpendicular side slopes, to many miles in nearly flat plains. The behavior of the river in flood stages is determined not only by the physical properties of the normal channel, but also by the physical properties of the flood plain.

Protection against the maximum flood which has occurred or is likely to occur is not considered feasible or desirable in some areas, particularly in small drainage basins.(4) Instead, protection is provided against a flood of a given frequency. The question of whether or not partial protection against floods is justifiable is beyond the scope of this paper. The fact remains, however, that some flood control projects are being designed and constructed for partial protection only. Therefore, a very important consideration in an analysis of a flood situation is the frequency with which floods of certain magnitudes are likely to occur. The frequency of floods is usually estimated by statistical analysis of the available data. The reliability of the analysis depends on the length of flood record, with respect to time, the accuracy of the data used, and the extent to which the flood frequency curve is extrapolated. The frequency and magnitude of floods, together with an analysis of the damages experienced during each flood and the estimated cost of protection for each flood, determine to a large degree the flood control design.

Economic Considerations

Until the advent of the motor age, it was economically advantageous for

many industries to be located in the flood plain.(5) Convenient water transportation, usable processing water, a ready and inexpensive method of disposing of wastes, and the low cost of land more than compensated for the damages resulting from occasional floods. Some of these reasons no longer have an economic advantage. Today, many industries are seeking locations, on or near major highways, which have a dependable water supply and attractive surroundings. The problem of getting to work is no longer a major consideration since, with the widespread use of the automobile and good roads, employees are not only able and willing to commute to work but prefer the wider choice of home sites this mobility gives them. However, some industries and utilities do have a valid reason for remaining in the flood plain.

Flood plains have generally been considered as the ideal location for sewage treatment plants and wells for water supply. These facilities are necessary for the continued existence of modern cities. Those industries which depend on water transportation for their supply of raw materials, and as a means to ship the finished product, regard the flood plain as the only feasible location for their operations. In these instances, structural adjustments and emergency measures would alleviate the extent of flood damages. On the other hand, telephone exchanges, hospitals, penitentiaries and essential industries definitely should not be placed in the path of any conceivable flood waters.

The rich alluvial soil in some flood plains is considered by some persons to mean that the flood plain was ordained to have an agricultural role. In other instances, the swampy conditions of the flood plain has led to the filling in or draining of these swamps to reclaim the land for industrial and residential use, a role which is encouraged by the growth of cities and increasing land values.

In some areas, the extensive use of the flood plain makes it more economical to provide protection to that area than to evacuate it. The protection, however, is only against a flood of a certain magnitude. There is no assurance that a greater flood will not occur causing considerable damage because the feeling of security generated by the flood control works reduces the precautionary measures which were taken before the flood control works were installed. This feeling of security is likely to occur regardless of whether or not partial or full protection against floods is provided. The existence of the protective works, in addition, usually encourages further use of the flood plain.

For most towns and cities, which have local flood problems, the economic value of the flood plain does not warrant extensive and expensive control structures. Frequently, it is more economical to develop a stream valley into a park rather than to enclose the stream in a limited-capacity culvert.(6) The flood plains adjacent to the stream, utilized for parks, playground areas, golf courses and other recreational activities either public or private enhance the beauty of the city and contribute materially to the welfare of its citizens. The damage experienced during flood periods is minor and usually can be taken care of with the regular maintenance force. However, park buildings and facilities subject to extensive damage should be placed in areas which will not be flooded. On most small streams, the flood plain is inundated less than one per cent of the time per year. It is doubtful that this minor inundation would affect the lawn areas to any extent if no oil or harmful chemicals were present in the water.

In determining the feasibility of flood plain zoning from an economic point

of view, the following items should be considered:

1) Existing and potential flood damages.
2) Cost of protection.
3) Benefits from protection.
4) Cost of evacuation.
5) Benefits from evacuation.
6) Allowable land use.
7) Community benefits.
8) Structural adjustments of existing encroachments.
9) Emergency measures such as flood warning services, rescue squad and elevating goods and machinery above possible flood heights.

Social Implications

In ancient times, the most favored locations for cities were those where conditions were suitable for agriculture and for transportation. The rich alluvial soil in the flood plains with the convenience of nearby water transportation satisfied these requirements. These cities have expanded, in most instances, to higher surrounding areas above possible flood heights. However, the flood plain location of the original portions of many cities has been and still is reflected in many of the social problems facing cities today.

Periodic flooding of residential areas in flood plains fosters the development of blight. The unrealistic economic values, the high cost of providing services, the deterioration of the areas due to unwholesome topographic and physical surroundings—all contribute to the conditions existing in many developed flood plains. The existence of some of these factors is evident in th difficulty in evacuating the area or in providing adequate protection to the area. This protection usually requires extensive and costly flood control works, the proportioned share of which is usually beyond the financial abilit of the people directly concerned. Therefore, the community, at large, is faced with three alternatives:

1) Do nothing to the area and continue to pay for the high cost of services required and to absorb the cost of flood damages.
2) Evacuate the area and retain it as public lands to be used for suitable functions.
3) Provide adequate flood control works to protect the area.

Protection of the Flood Plain

The extent to which flood plains are zoned determines the degree of floo damage prevention which will result. The minimum amount of regulation in flood plain prohibits land development which causes a construction of the natural stream channel. This is sometimes called the "channel-capacity"(3 regulation since the degree of restriction depends on the capacity of the cha nel to discharge a flood of a certain frequency. The design frequency depen either on an economical analysis of the flood damages and benefits from pro tection or on the degree of protection desired. Naturally, a flood greater th the design flood will inundate the adjacent flood plain. The maximum regula tion of the flood plain limits the use of the flood plain to purposes which do not suffer from the action of a flood or to purposes which, if damaged, caus

no serious inconvenience to persons or interests located outside the flood plain. This regulation is often referred to as the "land-use"(3) regulation, which makes possible complete prevention of flood damage when this is considered necessary or desirable.

Most of the existing flood plain ordinances in existence today are based on the "channel-capacity" regulation. The reasons for this are: (1) it is easier to determine, (2) its enforcement is simple, (3) legal precedents have been established, (4) the maximum amount of land is available to owners of flood plain property for use and (5) it directly affects only riparian owners. "Land-use" regulations are becoming more prevalent despite the restriction placed on the land. Often, however, complete restriction is considered unnecessary and some use of the flood plain is permitted provided certain safeguards are met.

Inherent in the establishment of any area as unsafe because of possible flooding is the implication that the areas which are not thus delineated are secure from possible damage from floods. The degree to which the danger of this implication exists depends on the criteria used in establishing the limits of flooding. Flooded areas delineated by past floods may or may not include the area which could be potentially flooded by a future flood. It is very possible that, as drainage basin characteristics and channel conditions are altered, a greater flood height than had been previously experienced may be expected. In fact, it is always possible to have a greater flood than any previous flood regardless of whether or not there is any topographic change within the basin.

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CONTENTS

DISCUSSION
(Proc. Paper 1041)

Discussion of
"SYNTHESIS OF RAINFALL-INTENSITY-FREQUENCY REGIMES"

by D. M. Hershfield, L. L. Weiss, and W. T. Wilson
(Proc. Paper 744)

D. M. HERSHFIELD,[1] L. L. WEISS,[2] AND W. T. WILSON,[3] A.M. ASCE.— We appreciate Mr. Sammons' complimentary reference to multi-graphical correlation and his suggested improvements. In the evolution of figure 1 the primary requirement was to estimate 2-year, 1-hour rainfall for localities lacking even daily rainfall data. The daily rainfall parameter was added to the rest of the diagram for application to localities having daily data. Before applying the diagram the writers tested it with independent data from many parts of the world, and noted that it had rather broad applicability with tolerable error.

In arctic and subarctic regions this diagram was not as successful as another, shown here in figure 3. The large proportion of snow in the mean annual precipitation of arctic regions does not contribute much to a rainfall index, and days-of-precipitation is a difficult observation to make in snow regions. The two parameters, mean annual precipitation and number of days of precipitation per year, combine to give a measure of rain per rainy day. In a large measure the daily precipitation parameter which the writers added to the diagram in figure 1 is redundant to these two parameters.

It has been observed that over much of the world the average ratio of 2-year, 1-hour rainfall to 2-year, 24-hour rainfall is about 0.40. This ratio may be 0.20 or lower in maritime regions having a low incidence of thunderstorms, while in continental regions where there is a high incidence of convective activity the ratio may exceed 0.60. Figure 3 shows the relationship between annual extremes of hourly and daily rainfall as a function of thunderstorm incidence. The diagram is based on data from stations in Alaska, Canada and northern United States.

To apply this diagram to localities having more than 25 thunderstorms per year, the diagram would have to be extended. The shape of the diagram may vary slightly from region to region. We believe that this type of diagram, where the required data are available, and derived for the pertinent climatic region will be at least as good, simpler, and possibly superior to the one the writers published in the paper.

While the foregoing discussion is not specifically a reply to Sammons' discussion, the writers' paper was presented as a progress report, and it is felt that recent progress is pertinent.

Coming, now, to some of Sammons' points: The writers have found that 1.13 is a better average factor for converting annual series to partial-

1. Weather Bureau, U. S. Dept. of Commerce, Washington, D. C.
2. Weather Bureau, U. S. Dept. of Commerce, Washington, D. C.
3. Meteorologist, Weather Bureau, U. S. Dept. of Commerce, Washington, D. C.

duration series 2-year, 1-hour values; the 1.09 factor was based on a relatively small sample of data and, of course, even at the 2-year level there is some subjectivity in most methods of fitting curves to partial-duration data.

In deriving the diagram that Sammons discussed, the writers were not concerned with the distribution of any of the parameters; they were dealing with a statistic, essentially the mean. The relationship of the 2-year value to the mean varies trivially among the several methods for analyzing annual series. In figure 1, no particular distributions were explicitly assumed. The scales used were largely a matter of convenience, with their inter-relationships fixed, of course, by the relationships of the data themselves. The empirical relationships of the figure are not dependent on subsequent discoveries as to the nature of the distributions of the variables used.

It is not clear how Sammons obtained "Weather Bureau ratios" for the last table of his discussion. In the paper the writers explicitly suggested 5-year and 10-year ratios to the 2-year value of 1.4 and 1.7 as provisional average values in lieu of data for computing them. Weather Bureau Technical Paper No. 25[1] shows these ratios for about 200 United States stations. There is considerable variation among stations, but it is uncertain how much of this variation can be ascribed to physical and climatic causes, and how much is sampling error.

Sammons raised a question about the writers' stated record length averaging 40 years with respect to a base period. To avoid secular trends it is a common practice to restrict the data to a common period. This means eliminating data from outside this period, and stations whose records do not occupy all of the period. This period is sometimes made rather short to avoid excluding too many stations. The writers felt that most of their records were long enough and for nearly enough the same period of time to be included. They felt that they would lose more by restricting the volume of data than by trying to get a homogeneous period. At present the writers are testing this issue, and at the same time are experimenting with the effect of duration on the frequency distribution. The writers might possibly find that the frequency distribution of daily amounts represents a different population than of hourly amounts, in some parts of the country.

The writers might expect to find for some localities that hurricane rainfall defines the regime of daily extremes, while thunderstorm rainfall defines the hourly regime. In other localities they may find that the same storms frequently provide both the hourly and daily extremes. The writers want to find out under what conditions the relationship of daily to hourly extremes is a climatic index, and under what conditions it is a physical relationship.

1. U. S. Weather Bureau, Department of Commerce, Weather Bureau Technical Paper No. 25, Rainfall Intensity-Duration-Frequency Curves for Selected Stations in the United States, Alaska, Hawaiian Islands and Puerto Rico, Washington, December 1955.

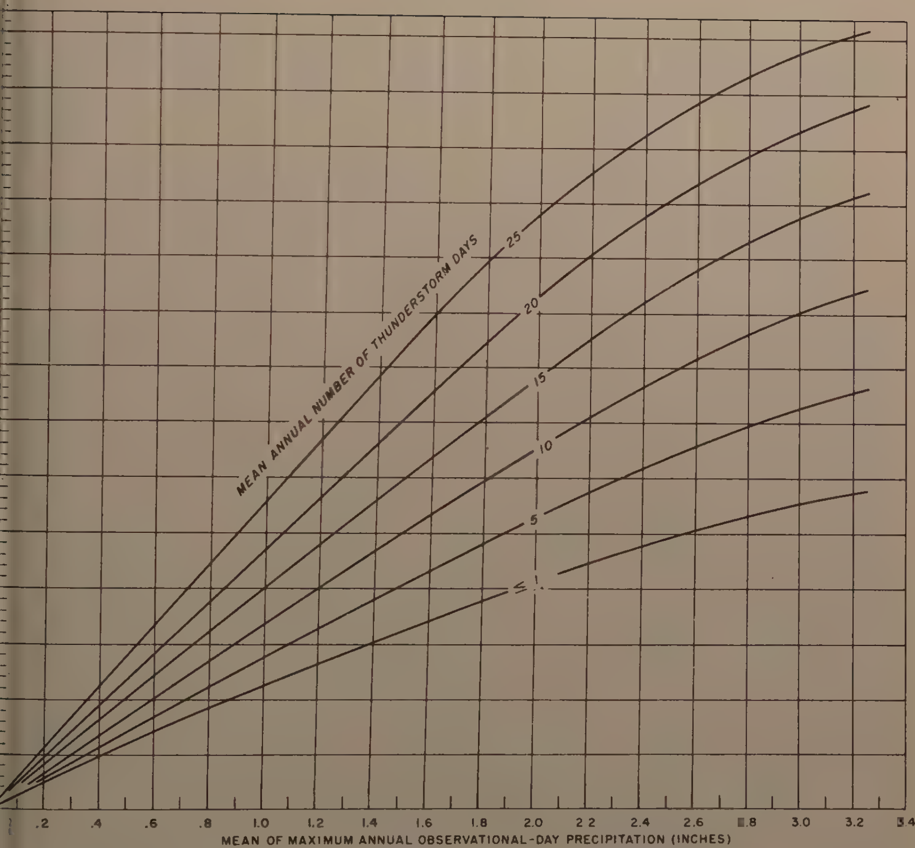

FIGURE ■ DIAGRAM FOR ESTIMATING 2-YEAR 1-HOUR RAINFALL

Discussion of
"GROUND WATER PHENOMENA RELATED TO BASIN RECHARGE"

by Paul Baumann
(Proc. Paper 806)

PAUL BAUMANN,[1] M. ASCE.—Mr. Lee correctly evaluates the writer's equation 1, although expressing the acceptance rate in c.f.s. per square foot instead of per wetted acre would have been preferable for the following reasons:

In the study of two-dimensional ground water movements a strip one foot in width is normally considered. Hence, the spreading grounds and the stream channels, the width of which rarely is constant, may be divided up into as many such strips as are required to reasonably reflect the effect of the variable width on the total acceptance rate.

In constructing the flow net Mr. Lee apparently was unduly influenced by the classical textbook example of percolation from a reservoir per unit length of a floating dam of infinite length on a saturated, porous foundation, with the less featured case of flow from a spreading ground under a river levee. The two cases are fundamentally different. The get-away of percolation from a saturated stream channel is approximately at right angle thereto, namely, in the direction of residual pore storage and of a control, as shown in the writer's Figure 3-a. In the longitudinal direction of the saturated stream channel there is no residual pore storage and no control and therefore no get-away for additional sub-surface flow. Hence, in the absence of lateral escape, all percolation under a floating dam from a reservoir must rise to the surface, whereas, percolation under a levee from an adjacent spreading ground must not so long as there is a get-away, by means of residual pore storage and a control. However, return flow to the river from the spreading ground may take place, depending upon the gradient between the water surfaces of spreading basin and stream channel as stated by the writer.

The flow net shown by Mr. Lee does not consistently satisfy the basic requirements therefor. These are: (1) the vertical drop Δh between the equipotentials along any phreatic line must be constant; (2) the flow Δq through each of the flow paths must be constant; (3) the flow Δq must be equal for all of the flow paths; (4) stream lines and equipotential lines must form quasi squares between themselves except near the boundaries where distortion is unavoidable. Not one of these conditions is reasonably satisfied by Mr. Lee's flow net. It therefore is unbalanced and cannot be accepted.

Fig. 11 shows a balanced flow net for the rather extreme condition similar to that chosen by Mr. Lee. The gradient between basin and stream channel water surfaces used in Fig. 11 is 2:1, as compared to the 5:1 gradient referred to by the writer as the limiting value for insignificant return flow to the stream channel. By assuming a number of phreatic lines through the levee it was

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possible to find the one for which the return flow is a maximum. For this condition the return flow was found to be 7.5% of the total flow toward the stream channel of percolation from the spreading basin. Hence, for a gradient of 5:1 the return flow would indeed be insignificant as stated by the writer. In arriving at the return flow to the stream channel, Mr. Lee computes the flow through two vertical sections on the premise that the respective gradients are equal to the difference in water surface elevation divided by the horizontal distance to the section. This is assuming horizontal flow throughout the entire reach in accordance with the Dupuit theorem. Under "Introduction" the writer pointed out that . . . "the Dupuit theorem is applicable except in special cases" . . . Subsequently, under the heading "Strip Basin" he stated, "It is apparent that the Dupuit theorem is applicable with fair approximation except near the spreading basin and near the control." There the flow conditions cannot be ascertained without a flow net. The total flow computed from the flow net checks that in accordance with the writer's eq. (1) within 15%, which is a close check considering the extremity of the example.

For the water year 1940-41 (October 1 to September 30) the rainfall for Los Angeles County was in excess of 200% of normal. Sustained runoff from the San Gabriel Mountains took place throughout the winter months and recharge of ground water basins through percolation from stream channels and spreading grounds caused mounds to rise to the surface in the "forebay" of the Central Coastal Basin by the Spring of 1941. Fig. 12 shows the ground water contours and ground surface contours in the vicinity of the San Gabriel and the Rio Hondo Spreading Grounds of the Los Angeles County Flood Control District as of May 6, 1941. These spreading grounds are located in the forebay. At stream channels the ground surface contours show average elevations between banks. Low water channel beds would be two to three feet lower. Of special interest is the merging of the mound beneath the San Gabriel Spreading Grounds with the channel mound, particularly in view of the fact that the get-away of percolation the east of the San Gabriel stream channel was more restricted than the get-away to the west of the Rio Hondo channel. Throughout this period there was no evidence of rising water in the saturated San Gabriel stream channel adjacent to the levee separating it from the spreading grounds, although the gradient from the spreading mound toward the stream channel is conspicuous in Fig. 12. Certainly there is no evidence of blocking by the channel mound of percolation from the spreading grounds. Of further interest is the curvature of the streamline trajectories normal to the ground water contours as referred to by the writer in connection with the channel flow net. A typical trajectory is indicated by a dash line in Fig. 12. It is hoped that this will also serve to clarify the question in regard to curvature of flow nets raised by Prof. Brooks.

Mr. Lee questions the similitude between the model test results and those of a prototype expressed by the writer's eq. (1). The model test which was relatively free from capillary flow[13] showed a maximum discharge parameter $q/K = 0.097$ whereas eq. (1) for $W = 2.4''$ (not $1.2''$ as used by Mr. Lee) and $D = 12''$ gives $q/K = 0.092$, a difference of 5-1/2%.

Prof. Brooks undertakes to correlate the evaluation of the writer's eq. (1) and his eq. (2) for the basic parabola. This is in keeping with fundamentally sound reasoning. However, in developing his eq. (2) so as to appear in terms of D/W Prof. Brooks apparently overlooked the fact that the depth D

13. Ibid, p. 1059.

in his Fig. A can never be equal to D in the writer's Fig. 3a. As previously[14] shown the phreatic line between the spreading ground and the control can be approximated by a double parabola. D in Prof. Brooks' Fig. A could therefore be considered as the depth below the point (P) of counter flexure, common to both, the lower and upper parabolas. Hence, for identification, D and L in Prof. Brooks' Fig. A should be changed to d and l. Prof. Brooks' example for arbitrary values of $D/W = 2$ and $L/D = 10$ and the apparent discrepancy between the results based on the writer's eq. (1) and on the basic parabola are therefore misleading because of the fact that for potential flow between the spreading ground and the control the ratio q/K must be common to both the spreading ground and the lower basic parabola. As explained by the writer, eq. (1) was developed from flow nets which necessarily terminated in basic parabolas. Hence, all corresponding values are found as the intersection of the l/d lines with the $(1 - e^{-d/W})$ curve in the writer's Fig. 13. It is interesting to note that the $l/d = 3/4$ line becomes tangent to the $(1 - e^{-d/W})$ curve as q/K approaches zero. In developing $e^{-d/W}$ into a series it also follows that as

$$d/w \rightarrow 0, \quad q/K \rightarrow d/W$$

Actually, as may be seen from Fig. 13 the q/K ratio closely checks the d/W ratio if the latter is less than 0.1.

In designing a spreading ground it is necessary to ascertain the economic limitations. Ordinarily, the depth $D = 2d$ to the impervious stratum is known. Hence, by means of the writer's eq. (1) and Prof. Brooks' revised eq. (2) corresponding values of q/K may be determined as shown in Fig. 14.

For the case quoted by Mr. Lee, namely, $D \sim 100$ feet, it follows from Fig. 14 that for $D/W = 0.10$ and therefore $W \sim 1000$ feet, the infiltration parameter per square foot $q/K = 0.05$ or $q = 3.35$ c.f.s. per wetted acre for a Meinzer Unit = 1000 as used by Mr. Lee.

Since Figs. 13 and 14 are correlated, the absolute value of l (which is half the distance between the edge of the spreading ground or stream channel and the control) may be ascertained from Fig. 13.

As previously stated, extraction of ground water through pumping is to be expected where recharge through spreading is performed. Coordination of the two operations should include the locating of water wells as closely as possible to the theoretical control. However, the curvature in plan of the flow paths, previously discussed and indicated in Fig. 12, mitigates the need for precise location of the control, based on two-dimensional flow. For, prototype ground water movement, contrary to that in models, is seldom strictly two-dimensional but can, within limits, be approximated as such.

Pumping in the spreading area proper, although effective in regard to infiltration, may not be desirable because of a short path of percolation and possible contamination especially in shallow ground water basins.

The dash line near the division of flow or axis of symmetry in the writer's Figs. 3a and 3b as questioned by Prof. Brooks is not intended to be a boundary. If it were, the equipotential lines would not extend across it. Hence, flow takes place along the vertical division line and the dash line merely meant to indicate the effective limit of the flow Δq in the flow path next to the boundary. Admittedly, the effective limit of flow could be modified to

14. Ibid, p. 1042-1045.

advantage as shown in Fig. 15, the symmetrical flow net for the free water surface of stream channel and spreading ground at the same potential.

Prof. Knapp, in presenting his interesting and constructive discussion has, indeed, found a flaw in the crystal ball in that it prophesied ultimate condi-dions when, in fact, they were just "prior to ultimate." Of particular significance is Prof. Knapp's reference to interface slopes under the influence of pumping. As may be judged from the writer's Fig. (4) the draw-down gradients adjacent to the test site are of the order of 1/2 of 1%. Hence, in using the 40 to 1 ratio as correctly suggested by Prof. Knapp, the slope of the interface would be of the order of 20%. This phenomenon is somewhat startling at first glance. However, normally sea water spread along the bottom of an aquifer is "sucked" into wells at moderate rates as compared to fresh water because of the lowest well perforations being at such a distance above the bottom of the aquifer as to require the interface to assume its maximum slope in rising to the well.

This clearly brings out the difference between the saline wave and sea water intrusion proper, questioned by Prof. Hotes and Dr. Harder, in that during the passing of the saline wave the lowest perforations of affected wells may be temporarily blocked, whereas no measure of protection will save a well from abandonment if sea water intrusion is permitted to continue unchecked. Also, those wells adversely affected by the saline wave might be temporarily used to extract sea water cut off from the ocean by the barrier. In more timely terms the dissipation of the cut off part of sea water intrusion may perhaps be compared with containing the cut off part of an invasion army as compared to containing the whole invasion army.

It was the writer's privilege to observe the ingeniously conceived model, shown in Prof. Hotes' Fig. 1, at the Richmond Field Station of the University of California. As is clearly brought out in Prof. Hotes' discussion, this model lent itself to simulate well-nigh all probable prototype conditions and to evaluate results quantitatively within reasonable limits of accuracy. As in all cases where ground water studies are concerned, the time scale would have to be used on the usual assumption of homogeneity of the aquifer as well as uniformity in magnitude.

Prof. Hotes' eq. (1) which was developed by Dr. Harder and which is an adaptation of an example of flow through an earth prism analyzed by Dr. Muskat, is based on the assumption of a constant potential or equipotential along a vertical line through the injection well; a constant potential of equal magnitude along the horizontal sole and a variable potential along the vertical line through the seaward control. Hence, the fresh water flow passes through the control between the bottom of the clay cap and the sole of the aquifer with no flow along the latter. Exactly the same result as obtained by Dr. Harder can be obtained by applying the Dupuit theorem to flow between the clay cap and the interface. The shape of the interface is such as to have balanced pressure between fresh water and sea water at all points. Its shape is expressed by

$$z = M \pm \sqrt{M^2 - \frac{2qx}{K(s-1)}} \quad \text{------ (2)}$$

in which, as shown in the writer's Fig. 16

z = Ordinate of interface above sole (x - Axis)

M = Magnitude of aquifer
x = Abscissa relative to center of recharge well
q = Rate of flow per unit width of aquifer
K = Permeability coefficient
s = Specific Gravity of sea water
1 = Specific Gravity of fresh water.

For any stable condition of the interface, the rate of flow q must be constant and a maximum. Hence, because the radical in the writer's eq. (2) must be real, the two terms under the radical for q_{max} must be equal. Thus for x = L

$$M^2 = \frac{2\,qL}{K(s-1)} \quad \text{- - - - - - - - - - - (3)}$$

and $$q = \frac{1}{2}(s-1)\frac{KM^2}{L} = \frac{1}{2}(s-1)\frac{M}{L}\,T \quad \text{- - - (4)}$$

which checks Prof. Hotes' eq. (1).

It now follows that for the above condition to obtain, the product qL of the two variables must remain constant. Hence, as q increases, L decreases proportionately and vice versa as is evident from Prof. Hotes' Fig. 3. From the writer's eq. (2) it would follow that for any stable condition of the interface and therefore any corresponding product qL, the height z of the interface at the control is equal to M. As this would leave no aperture for the discharge of fresh water, it is evident that eq. (2), based on the Dupuit theorem, is an approximation of the true solution. To satisfy z = o for x = o the sign before the radical must be negative.

From the writer's eq. (2) it follows that the slope of the interface

$$\frac{dz}{dx} = \frac{q}{K(s-1)\sqrt{M^2 - \frac{2qx}{K(s-1)}}} \quad \text{- - - - - (5)}$$

and that for x = L the slope for any corresponding q and L values is 90^o. While this answer again reflects the afore-mentioned approximation, it appears to be reasonable if we remember Prof. Knapp's 40 to 1 ratio between the fresh water gradient and the slope of the interface. In accordance with this ratio, any fresh water gradient in excess of 2-1/4 degrees would call for a slope of the interface at the control of 90^o which, of course, cannot be exceeded.

If the interface were treated as an inverted, basic parabola and if the boundary condition z = M - q/K at x = L were imposed to bring the Dupuit solution in accord with the Kozeny[15] solution, then the writer's eq. (2) would lead to

$$q/K = -\frac{L}{(s-1)} + \sqrt{\frac{L^2}{(s-1)^2} + M^2} \quad \text{- - - - (6)}$$

15. Ibid, p. 1040.

and $$q = -\frac{KL}{(s-1)} + \sqrt{\frac{K^2L^2}{(s-1)^2} + T^2} \quad \text{----------} (7)$$

While no authenticity is claimed for eqs. (6) and (7), it is encouraging to find that the boundary conditions $L = o \therefore q = T$ and $L \rightarrow \infty \because q \rightarrow 0$ are satisfied. Also that q is of the same order as that from eq. (4).

Putting eq. (6) in the form $q/K = \frac{L}{S-1}\left(\sqrt{1 + \frac{M^2}{L^2}(s-1)^2} - 1\right)$

and expanding the radical into a binomial results in

$$q/K = \frac{M^2(s-1)}{2\,L} - \frac{1}{8}\,\frac{M^4\,(s-1)^3}{L^3} \quad \text{-------------} (8)$$

from which it follows that for any finite discharge face the rate of flow q must be smaller than in accordance with Prof. Hotes' eq. (1). Hence, the latter equation should be corrected to read

$$q = \frac{K\,(s-1)\,M^2}{2nL} \quad \text{-------------} (9)$$

in which $n > 1$ and increases as the ratio $\frac{M}{L}$ increases.

Obviously, if fresh water from the recharge well were discharged from a pressure level, h feet above M.S.L. at the recharge well into a fresh water lake with the water surface at M.S.L. and if the interface were fixed, y_o at the control would be close to zero and the pressure gradient or piezometric line would be as indicated in Fig. 16. If now it were possible to suddenly con- vert the back pressure due to fresh water to that due to sea water, the dis- charge would stop just as suddenly. But the pressure head h would now exert itself and would cause fresh water pressure to exceed sea water pressure ex- cept at the interface where the two pressures would be balanced. Hence, for a liquid interface the blocking of flow at the base of the control is offset, at least in part, by the pressure head y_o on top of the control.

This moves into focus the danger of "blow-out" at the control which grows with q and diminishes with the increase in the distance L between the toe of the salt water wedge and the control. Furthermore, this danger is greatest during periods of extreme low tide, a feature particularly significant at loca- tions subject to large tidal ranges. Fortunately, the extreme tidal range along the coast of California is moderate.

It is possible that offset in the model, shown in Prof. Hotes' Fig. 1, be- tween the back of the sea water weir box and the control face gave rise to the trapping of air split out of the fresh water and that the q measurements were thereby affected to some extent.

Of considerable interest would have been the determination of the equipo- tential lines and therewith the flow distribution between the interface and the "clay cap" of the model. This is just one of many facets meriting additional research.

The discharge through the control[16] to the ocean of fresh water could be more readily comprehended if the interface, as a basic parabola, were extended so as to terminate at the horizontal discharge face, q/2K in width and flush with the bottom of the clay cap. The discharge of fresh water would then be vertically upward and would thereby satisfy the physical premise of buoyancy of fresh water relative to sea water.

Of special interest are the 10 stages shown in Prof. Hotes' Fig. 4 and particularly those pertaining to high injection rates. It must be assumed that the condition as shown at the bottom of the right-hand pictures would represent close to the ultimate condition of the sea water wedge cut off by the barrier.

Dr. Harder computes the contamination of 240,000 acre feet of water to a concentration of 250 parts per million of chloride ions (Cl^-) because of diffusion of that part of the sea water wedge which had intruded landward from the injection wells. If sea water in intruding a fresh water aquifer mixed as readily with the fresh water through diffusion as Dr. Harder seems to assume, the correction of sea water intrusion would indeed be a next to hopeless case unless it were always possible to locate the injection wells landward from the extreme tip of a saline wedge as suggested by both Prof. Hotes and Dr. Harder. At Manhattan Beach this procedure was carefully considered but had to be abandoned because of three principal features. First, the landward pressure gradient of the aquifer would have made it necessary to create a barrier above mean sea level of approximately twice the height of that at the location where the barrier was finally located; second, the right of way necessary for the barrier including pipe lines, pressure regulators, chlorinators, etc., would have been far too expensive to be justified in the light of an experimental period of operation; and third, facilities for the disposal of water produced during development of the recharge wells were completely lacking. However, the writer still maintains that the effect of the saline wave, under similar conditions as those at Manhattan Beach, is negligible as compared to that of the sea water intrusion proper. It is only necessary to visualize the irregularities normally existing along the bottom of an aquifer in the form of mounds and depressions to realize the facilities for ready absorption of the sea water cut off from the original wedge by the injection and forced landward. The average depth of sea water so absorbed may readily be such as to require relatively short landward distance of spread.

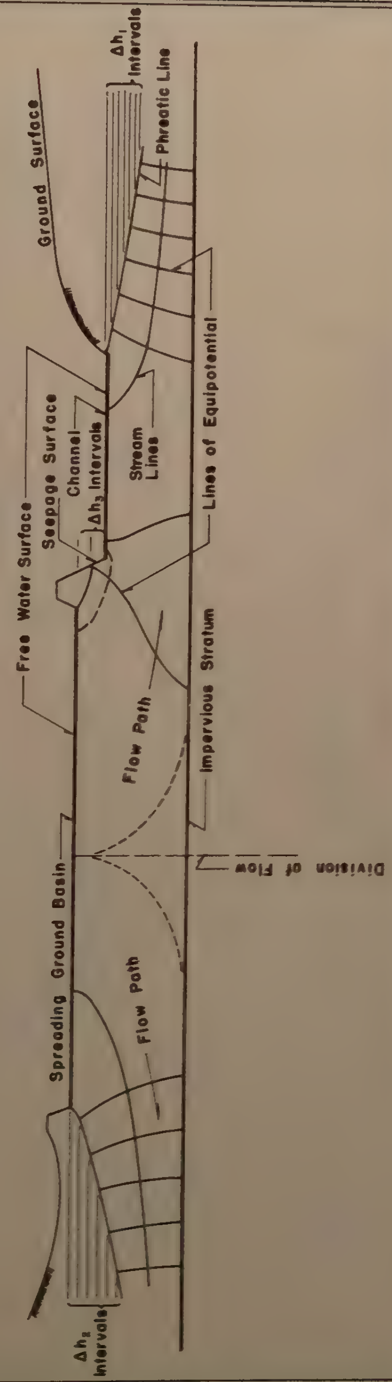
In referring to the writer's Fig. 8, Prof. Hotes concludes that the saline wave is worse than indicated by the writer. Apparently, Prof. Hotes overlooked the significance of the writer's Fig. 7 which shows more tellingly than Fig. 8 what happened when the cut off sea water wedge was forced landward by the barrier wells. For example, it shows that the salinity in well G-8, 1180 feet from the line of recharge wells dropped from about 5000 p.p.m. of Cl^- concentration to approximately 1000 p.p.m. in 16 months of barrier operation. Since October of 1955 this particular well has been immersed in fresh water of less than 250 p.p.m. of Cl^-. Prior to commencement of the recharge test, sea water displaced fresh water throughout the entire depth of the aquifer at a rate of some 500 feet a year. Hence, in the three years and two months since the test was started unadulterated sea water would have overtaken well G-8 and would have advanced some 400 feet beyond it. The writer's "optimism," therefore, does not seem to be entirely unjustified.

16. Ibid, p. 1045.

Dr. Harder and Prof. Hotes' points are however quite well taken in that sea water that had intruded into a shallow aquifer for a considerable distance landward and had completely displaced the fresh water for such a distance could not be checked by means of injection wells without seriously affecting close-by inland wells by the displacement of the landward part of the saline wedge. On the other hand, wells close to the barrier would, under conditions similar to those at Manhattan Beach, be close to the ocean as well and therefore subject to permanent and irreparable instead of temporary damage.

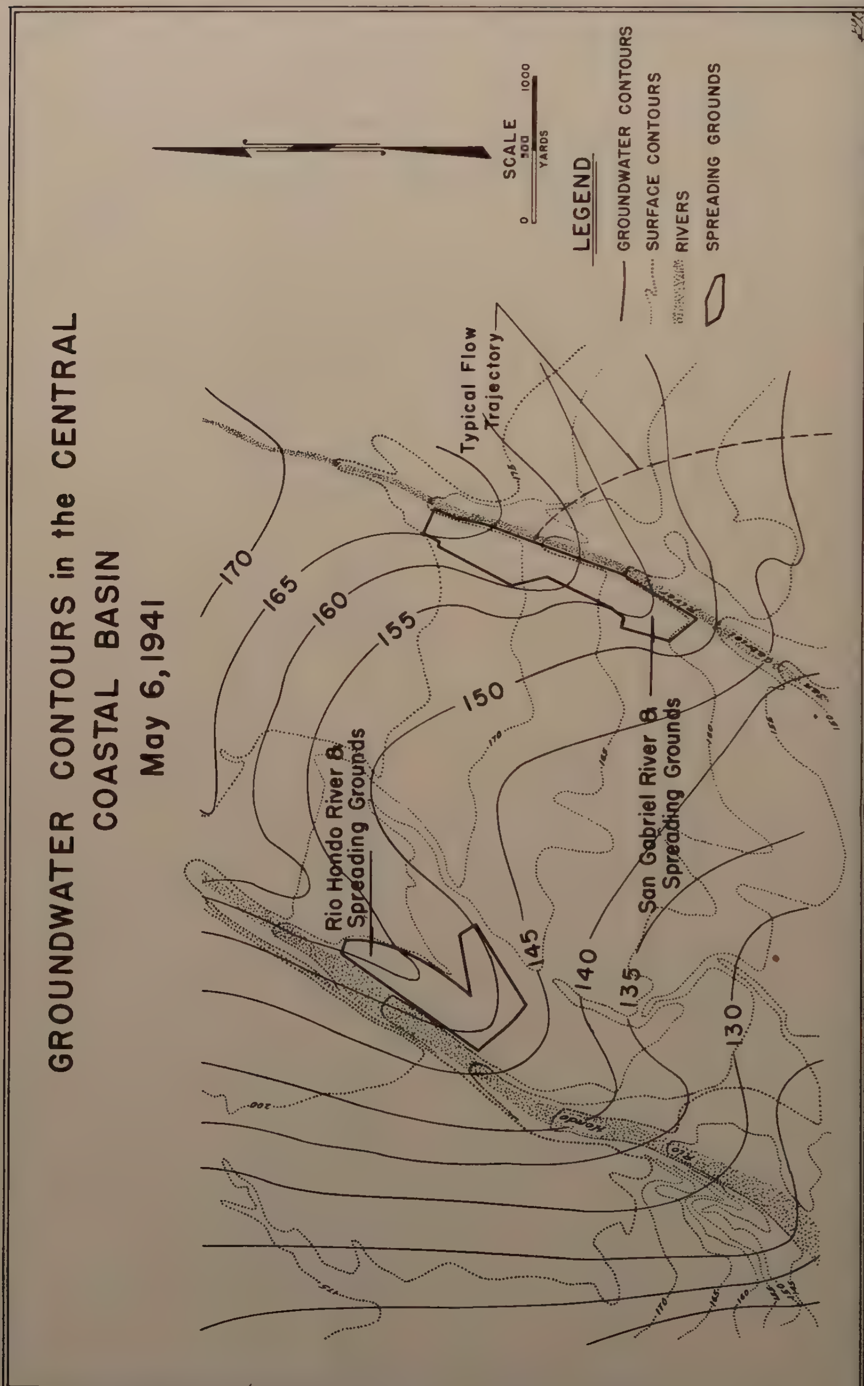
The model tests conducted by the University of California at Richmond have certainly paid dividends far beyond the return that could reasonably have been expected.

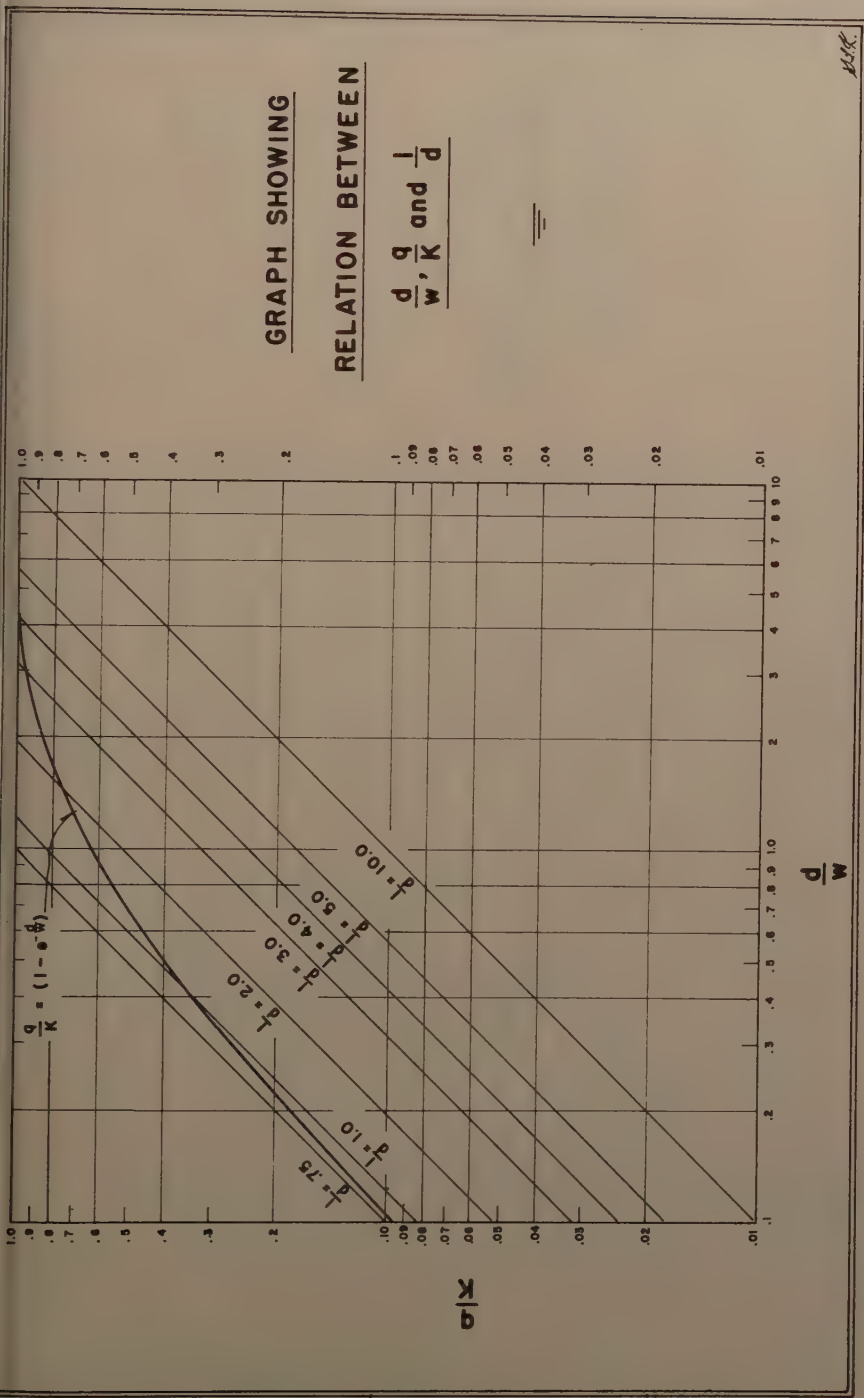
The writer is deeply appreciative of the spirited discussions through which great value was added to his paper. He only regrets his inability to agree with all of the thoughts therein presented.

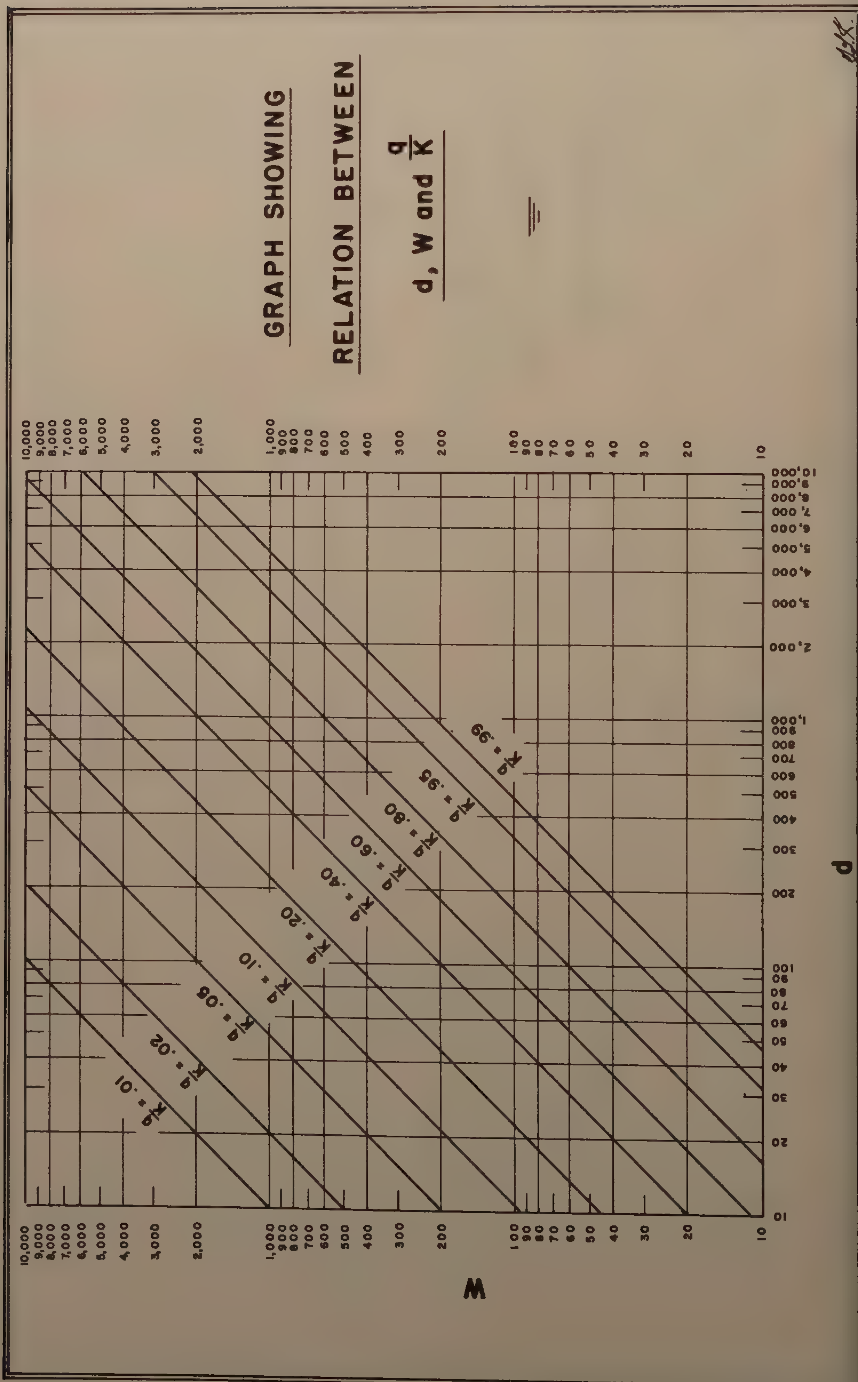


SECTION SHOWING COMBINED SPREADING GROUND AND CHANNEL MOUND FLOW NET

when $D/w = 0.1$


GROUNDWATER CONTOURS in the CENTRAL
COASTAL BASIN
May 6, 1941
SCALE
0
500
1000
YARDS
LEGEND
GROUNDWATER CONTOURS
SURFACE CONTOURS
RIVERS
SPREADING GROUNDS
Typical Flow Trajectory
Rio Hondo River & Spreading Grounds
San Gabriel River & Spreading Grounds
170
165
160
155
150
145
140
135
130


GRAPH SHOWING
RELATION BETWEEN
d/w, q/K and l/d
q/K = (1 - e^(-d/w))
l/d = 10.0
l/d = 5.0
l/d = 4.0
l/d = 3.0
l/d = 2.0
l/d = 1.0
l/d = .75
d/w
q/K


GRAPH SHOWING
RELATION BETWEEN
d, W and q/K
q/K = .01
q/K = .02
q/K = .05
q/K = .10
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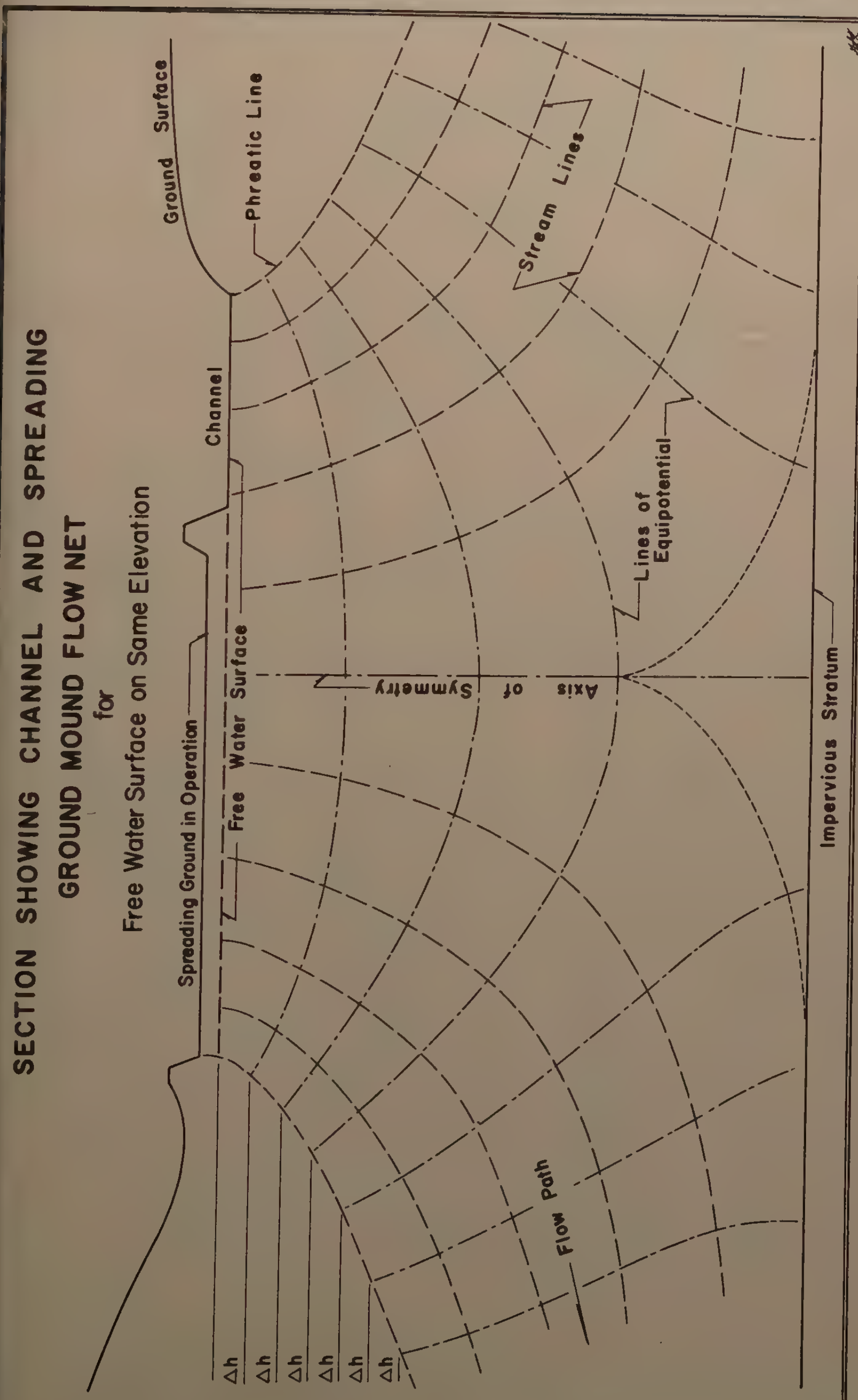
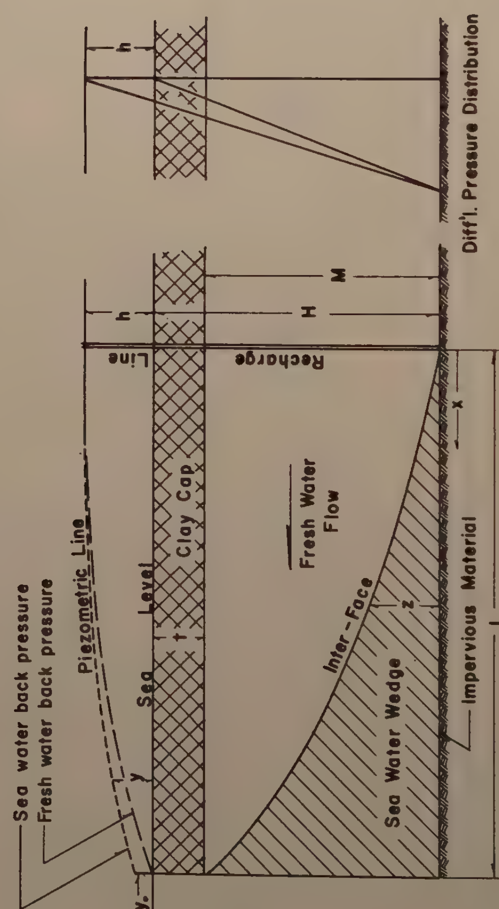
q/K = .95
q/K = .99
d
W



SECTION SHOWING CHANNEL AND SPREADING GROUND MOUND FLOW NET for Free Water Surface on Same Elevation



WEST BASIN BARRIER TEST

DIAGRAM OF SECTION THROUGH PRESSURE AQUIFER

Discussion[1] of
"EXTENDING STREAMFLOW DATA"

by W. B. Langbein and C. H. Hardison
(Proc. Paper 826)

F. A. JOHNSON,[2] A.M. ASCE.—A graphical method is presented for estimating the relationship between monthly discharges of nearby streams having similar characteristics. The relation curve then is to be used to extend the shorter record on basis of the other, or long-term record; and statistical measures are suggested as an indication of the expected accuracy of the estimates.

Several experiments made by the writer with pairs of long-term records led to fairly reliable extensions in some cases, but not in others. The statistics, such as standard error of the relation curve and the coefficient of correlation, did not invariably give a dependable indication of the accuracy.

It was apparent from these tests that in some regions the areal pattern of runoff was relatively stable from year to year, and over long periods. In other regions, however, the distribution varied greatly from year to year, or between irregular periods of several years. These instabilities or changes in pattern of runoff were noticed in a number of regions, both of the Pacific and Atlantic Slope basins. They apparently were caused by variations in the areal distribution of precipitation.

An average relationship between the monthly discharges of two nearby streams, say for 5 years of concurrent records, thus may differ considerably from that during another period, and lead to systematic errors in the estimates. An obvious way to minimize such errors, as well as the random errors, is to establish a relation between the discharge in the subject or satellite basin and the average discharge from two or more nearby basins on different sides of the one under consideration. For correlation of monthly discharge, however, this possibility often is limited by lack of sufficient long-term records having similar characteristics as to monthly distribution.

One purpose of this discussion is to describe a method for estimating annual or long-term runoff of the satellite stream from two or more long-term base records, regardless of the monthly distribution of runoff. In many parts of the Atlantic and Pacific Slope basins this leads to fairly accurate estimates, which may be compared with extensions of monthly records on an annual basis. Such comparisons are, in effect, independent tests of the accuracy of the extensions, if the annual estimates are based on different index records; and they sometimes may be used for purposes of adjustment.

The need for such a test is demonstrated by an example of the authors' correlative estimates which is summarized in Table 1 and 2, and Figure 4. In a number of similar experiments it was found that substantial errors of

1. Publication authorized by the Director, U. S. Geological Survey.
2. Regional Hydr. Engr., Geological Survey, U. S. Dept. of the Interior, Tacoma, Wash.

the kind illustrated here were not uncommon, although generally of somewhat lesser magnitude.

In this example, the monthly runoff of the Wading River near Norton, Mass. was estimated from that of the Taunton River at State Farm, Mass. The drainage areas are 42.4 and 260 square miles, respectively. The center of the Wading River basin is about 15 miles east of that of the Taunton River basin. Both basins are near the Atlantic Coast in the southeastern part of Massachusetts. A relation curve was defined on logarithmic paper by 60 pairs of observations recorded from October 1945 to September 1950. (Runoff figures in inches, as adjusted for diversions and storage, were readily available and were used without allowance for differences in length of months). The graphical method illustrated by the authors in Figure 1 was used for fitting the curve and estimating the statistical measures. The standard error of estimate was determined as 0.10 log unit, and the coefficient of correlation as 0.975.

Estimates made for the period 1930 to 1945 from this curve are summarized by annual totals in Column 2 of Table 1 for comparison with the recorded figures of Column 3. The runoff for two 4-year periods was underestimated by about 20 percent, which is approximately the standard error as determined for monthly estimates. The relationship during the sample period, 1946 to 1950, evidently was not representative of other periods between 1930 and 1945.

The authors pointed out that the statistical measures estimated from the sample are applicable in other periods only if the mean line is correctly located. However, it is difficult to see how this can be judged from internal evidence. It was pointed out also that these measures have the usual significance only when deviations from the mean line have a normal distribution. Although adjustments such as the logarithmic transformation often appear to bring the deviations into a normal distribution, the deviations may resemble random variations only if regarded without relation to time. The writer has noticed that large positive or negative deviations of monthly values sometimes persist for periods of from 4 to 6 months. These consistent deviations sometimes result in an accumulation of error in the annual totals almost as large as the percentage errors of the monthly estimates. Because of such errors it is evident that the computed standard error of the relationship may be an underestimate even for the sample period. It may be grossly in error during other periods if there is a bias of the kind illustrated in Table 1 and Figure 4.

It was noticed also that the seasonal distribution of discharge of some streams of the Pacific Slope is such that the monthly values tend to cluster in zones above and below the mean and median. Under these circumstances the coefficient of correlation does not have the same significance as for a normal distribution, and is apt to be fictitiously high. Coefficients of more than 0.99 in fact did not appear to be uncommon.

The recorded annual runoff for the example of Table 1 is shown in Figure 4. As would be expected, the annual values of runoff computed from monthly estimates fall approximately on an average curve through the recorded annual figures for the years used in definition of the monthly relationship. The degree of correlation between annual discharges of long-term records in a given region thus furnishes an indication of the reliability to be expected of monthly estimates for other streams in that region. Such comparisons would seem to be a useful preliminary step in extensions of streamflow data such as proposed by the authors.

It has been the writer's experience that the possibility of large systematic and random errors can be minimized most effectively by establishing a relationship between the subject stream and the average of several nearby streams, rather than between two streams alone. Along the Pacific Coast correlations seem to be the most reliable where the index basins are on opposite sides of the subject basin, and on a line roughly normal to the west-east, or average direction of the storm paths. A similar condition may exist in other regions. In connection with their discussion of snow-fed streams, the authors mentioned the possible need, at times, for use of more than one index of discharge to improve the correlation. It would seem that such precaution should be taken as a matter of course, since it can seldom be known that a group of observations in one period is closely representative of conditions in another period.

If the monthly distribution of runoff in the several basins is similar, the average of several records may be used in the same way as was illustrated by the authors with a single index record. However, where there are substantial differences between the altitude characteristics of the basins, and runoff results both from rainfall and snow melt, the monthly distribution may differ considerably. In that event, average indices of annual runoff may be used for defining relationships of annual runoff. Estimates of annual runoff then will serve to verify the accuracy of annual sums of monthly figures estimated in the way proposed by the authors.

Although the error due to variations in the areal distribution may be minimized by using averages, a relation line defined by only a few pairs of annual values obviously may not be dependable for purposes of extension. Fortunately, in humid regions where the precipitation is substantially more than the normal evapotranspiration, fairly reliable estimates usually can be made by a computed relationship described in the following paragraph. This has been used by Knox and Nordenson(1) for studies in the northeastern part of the country, but its wider applicability may not be generally recognized.

Ignoring systematic and random variations in water loss, and changes in natural storage, the relationship between the annual precipitation on nearby areas can be expressed as follows:

$$R_y + \overline{L}_y = b(R_x + \overline{L}_x),$$

where R_y and R_x are values of annual runoff, $\overline{L}_y$ and $\overline{L}_x$ are estimated values of mean annual loss, and b is the ratio of precipitation on basin "y" to that on basin "x." Both terms of the equation are subject to error equal to the deviation of the actual losses from the assumed average losses, plus the changes in natural storage. However these errors tend to be the same for nearby areas, and thus tend to cancel out. The ratio, b, often can be estimated closely from the average of only a few pairs of annual observations, even if their range of magnitude is small or negligible, so that a relation curve could not be defined graphically. The relation, $R_y = b(R_x + \overline{L}_x) - \overline{L}_y$ then can be used to estimate values of R_y from R_x during periods of missing record. The results usually are affected only slightly by fairly large differences in the assumed values of average loss, $\overline{L}_x$ and $\overline{L}_y$, from actual values, or by variations in the amount of annual loss with variations in precipitation. Information regarding natural water losses can be obtained from a number of sources. Some data were compiled by Rafter(2) as early as 1903, later by

Williams and others[3] in 1940, and by Lee[4] in 1942.

This method of estimation is illustrated in Table 2, as a supplement to Table 1. (In this example the ratio, b, happens to be 1.00. The method seems to work just as well where there are considerable differences in the values of areal precipitation). The annual estimates of Table 1, as well as for a number of unlisted experiments, are in general substantially more accurate and consistent than totals derived from monthly correlations between two streams alone. The improvements were due to the use of more than one index record for the annual estimates. The same improvement in monthly estimates could be obtained by this means, where the monthly distribution is closely similar in all of the basins.

In an actual extension of records, the reliability of the estimates can be judged only on basis of the errors for the years of definition, or by comparison with estimates made from independent correlations with other groups of records. The standard error of annual estimates computed for only a few years of record obviously may be subject to considerable error, but it at least affords a crude measure of the probable magnitude of error if bias has been substantially minimized. Since relationships between annual figures can be established rapidly, this provides a convenient means for selection of groups of index records that probably will be the most reliable for extension of monthly records in the way proposed by the authors.

This method of analysis was applied to the example given by the authors for estimation of the discharge of Murder Creek near Evergreen, Alabama from that of Sepulga River. As shown in Table 3, there was a better correlation between the annual runoff of Murder Creek and the average of two streams, Sepulga River and Escambia Creek, than between that of Murder Creek and Sepulga River alone. The Escambia Creek basin is on the other side of Murder Creek, about 30 miles to the southwest. (The indices for the Escambia Creek basin were multiplied by a constant factor, 0.88, to avoid giving greater weight to the variations in that basin than to the variations in the Sepulga River basin, because of the relative magnitude of the figures). Since the monthly distribution of the runoff in the three basins is similar, the comparison of the standard errors for the period of definition, 1941 to 1945, suggests that a substantial improvement in the monthly correlation also could be obtained by use of average indices. The relationship between the monthly discharge of Murder Creek and the average of the other two streams (in amounts per square mile) in fact is nearly a linear one for the entire range, and the standard error for the sample of 60 values is only 0.05 log unit, or about 60 percent of the authors' value. This improvement is roughly the same as between the two sets of annual estimates listed in Table 3, for the years 1941 to 1945.

Statistical tests, whether of monthly or annual values, will not show whether the relationship for the period of definition is biased in relation to that for another period, as in the example of Figure 4. For that reason it would seem desirable to compare the results of monthly estimates with annual estimates based on the average of several index records whenever the monthly indices themselves cannot be averaged to minimize the possibility of bias.

If it appears that estimates of annual runoff for a given stream are apt to be more reliable than estimates of monthly runoff, the latter could be multiplied each year by the ratios of the annual estimates to the sum of the monthly estimates. It would have been desirable to make such adjustments in the

estimates of Table 1, for example. This would have brought the estimates of mean runoff for the 4-year periods, 1935-1938 and 1942-1945, from within roughly 20 percent to within a few percent of the correct values. If the periods happen to be critical ones from the standpoint of dependable water supplies or regulation by storage, such improvement would be of considerable importance.

The graphical method proposed by the authors for estimating the relationship between monthly values of runoff is simple and can be applied rapidly. It is much more practical than numerical analyses, and considering the possible magnitude of sampling error in short-term correlations, more elaborate methods of analysis usually would not seem appropriate. The writer agrees that extension of short-term records is of growing importance and that the suggested methods constitute a useful approach to a sound procedure for extending records of monthly discharge. It is believed, however, that the statistical measures should be viewed with caution as indices to the reliability of such estimates and that there is a need for independent estimates and controls of the kind proposed in this discussion.

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Table 1

Estimated runoff of Wading River near Norton, Mass., compared with recorded runoff.

(1)	(2)	(3)	(4)	(5)	(6)
	Runoff, inches a/		Error in % of estimated values	Runoff, inches b/	Error in % of estimated values
Water Year	Sum of monthly estimates	Recorded		Annual estimates	
1930	13.4	12.6	+ 6.3	13.5	+ 6.7
31	20.4	23.7	-16.0	24.5	+ 3.3
32	15.5	15.1	+ 3.1	17.8	+15.2
33	31.0	29.3	+ 5.6	31.4	+ 6.7
34	21.2	23.2	- 9.6	23.1	- 0.4
1935	17.2	23.5	-36.6	24.2	+ 2.9
36	20.6	25.3	-22.3	24.4	- 3.7
37	24.1	24.8	- 3.2	26.0	+ 4.6
38	27.1	33.8	-24.7	33.7	- 0.3
39	24.2	23.9	+ 1.5	26.2	+ 8.8
1940	20.7	22.0	- 6.3	22.4	+ 1.7
41	15.4	14.8	+ 3.9	15.7	+ 5.7
42	11.5	16.1	-39.9	15.3	- 5.2
43	17.7	21.2	-19.8	21.5	+ 1.4
44	9.7	11.8	-21.8	13.1	+ 9.9
1945	22.0	27.7	-25.9	25.2	- 9.9
Mean	19.5	21.8	-11.8	22.4	+ 2.7

a/ Monthly estimates were based on the runoff of Taunton River.

b/ Annual estimates were based on arithmetical averages for Taunton River at State Farm, Mass., Charles River at Waltham, Mass., and Blackstone River at Woonsocket, R. I. The relationship of the indices to the runoff of Wading River was established for the period, 1946-50, by the method illustrated in Table 2. The Charles basin (227 sq. mi.) is about 20 miles north of Wading River, and the Blackstone basin (416 sq. mi.) is about 25 miles northeast.

Figures of recorded runoff were taken from Water-Supply Paper 1301 and rounded to the nearest tenth in this table.

Table 2

Annual runoff of Wading River as estimated from relationship to indices of areal precipitation; 1946 to 1950

All data in inches

(1) Water Year	(2) R_1	(3) R_2	(4) R_3	(5) R_x	(6) R_x plus 25.0	(7) Col.6 times 1.00	(8) Col.7 minus 25.0	(9) R	(10) Error dy
1946	33.1	26.5	26.2	28.6	53.6	53.6	28.6	29.6	-1.0
47	19.9	17.1	18.8	18.6	43.6	43.6	18.6	16.7	+1.9
48	32.8	23.7	24.9	27.1	52.1	52.1	27.1	29.6	-2.5
49	22.3	15.4	16.6	18.1	43.1	43.1	18.1	18.6	-0.5
1950	12.4	13.6	15.7	13.9	38.9	38.9	13.9	11.6	+2.3
Total	-	-	-	106.3	-	-	106.3	106.1	+0.3
Estimated loss:				125.0				125.0	
Estimated precip.				231.3				231.1	

Annual runoff (adjusted for diversions and storage)

R_1: Taunton R. at State Farm, Mass.

R_2: Charles R. at Waltham, Mass.

R_3: Blackstone R. nr. Woonsocket, R.I.

R_x: $(R_1 + R_2 + R_3)/3$

R : Wading R. nr. Norton, Mass.

Ratio, b: 231.1/231.3 = 1.00

Standard error of estimate: $\sqrt{\sum dy^2 /(n - 1)}$ = 2.02 inches

Estimates made for the period 1930 to 1945, as in column 8, are listed in Table 1. The standard error during this period was 1.40 inches.

Note: Columns 6, 7 and 8 are unnecessary in this example, but are shown to illustrate the usual procedure.

Table 3

Annual runoff of Murder Creek near Evergreen, Ala., estimated from indices of areal precipitation.

(All data in inches)

Water Year	P_1	P_1'	P_2	P	R_y	R	d_y	R_y'	d_y'
1941	48.4	42.6	43.0	42.8	15.5	17.0	-1.5	15.8	-1.2
42	57.8	51.0	49.9	50.4	23.5	21.3	+2.2	23.1	+1.8
43	56.9	50.1	49.3	49.7	22.9	22.6	+0.3	22.5	-0.1
44	68.4	60.3	65.5	62.9	36.8	35.4	+1.4	39.9	+4.5
1945	53.6	47.3	42.3	44.8	17.6	19.9	-2.3	15.1	-4.8
46	66.2	58.4	68.0	63.2	37.1	36.2	+0.9	42.5	+6.3
47	62.3	54.9	53.4	54.2	27.6	28.4	-0.8	26.9	-1.5
48	60.5	53.4	50.8	52.1	25.3	25.5	-0.2	24.1	-1.4
49	69.8	61.5	62.8	62.2	36.1	33.2	+2.9	36.9	+3.7
1950	49.3	43.5	37.4	40.4	12.9	14.1	-1.2	9.8	-4.3
Mean					25.5	25.4	+0.1	25.7	+0.3

Average loss: 30 in./yr. (estimated)

$b = (R+150)/P = 1.062$

$b' = (R+150)/P_2 = 1.065,$

where values of P and R are totals for period, 1941-45.

$Ry = 1.062\,P - 30$

$Ry' = 1.065\,P_2 - 30$

P_1 = Annual runoff of Escambia Crk. + 30 inches

$P_1' = 0.88P_1$

P_2 = Annual runoff of Sepulga River + 30 inches

P = Average of P_1' and P_2

R = Annual runoff of Murder Creek

dy and dy' = Errors of computed values, Ry and Ry'.

Standard error of estimate

Estimated from deviations dy

1.90 inches, 1941-45

1.69 inches, 1946-50

Estimated from deviations, dy'

3.46 inches, 1941-45

4.35 inches, 1946-50

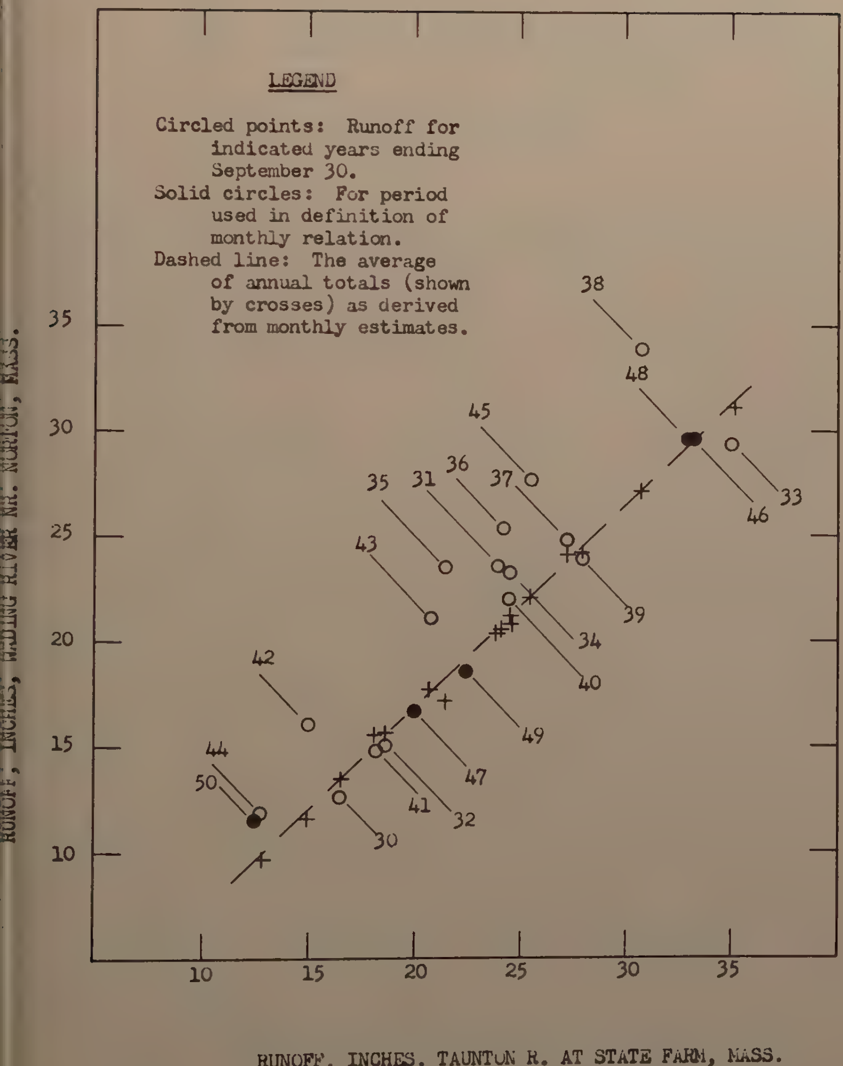

FIGURE 4 CORRELATION OF ANNUAL RUNOFF
October 1929 to September 1950

Discussion of
"FLOOD PLAIN ASPECTS OF RIVER PLANNING"

by Anthony M. Lunetta
(Proc. Paper 1040)

ROBERT L. SMITH,* A.M. ASCE.—The role of the flood plain in river planning is seldom limited to a specific type of water problem, nor is it always limited to consideration of just the water resource. Under certain conditions problems of land, wildlife, or mineral resource management can be expected to influence materially the planning decisions governing water resource development. By contrast the remarks in the foregoing paper have been limited to a specific water problem—floods. Likewise the remarks on the possible ways to minimize the damages caused by those floods were limited to the proposal that the use of land within the flood plain be subject to regulation. Further, those remarks were summarized in a very logical sequence and they set forth quite adequately the factors to be considered. Notwithstanding those various limitations, the most significant feature developed in those remarks was the acknowledgment of the breadth and, in essence, the complexity of the problem discussed. Of particular importance is the concept that this problem of wise utilization of the flood plain is interdisciplinary in nature. In fact, engineering consideration of the problem cannot dictate the answer, but can only serve to crystallize the solution to fundamental economic, legal, and social questions.

The subject presented is important. In every area of the country, in areas both developed and underdeveloped, day by day expansion is continually increasing the potential flood damage. The author suggests that this expansion is twofold. First, there is encroachment which is motivated by economic desire to utilize the resources of the flood plain. Secondly there is encroachment which did not seek the flood plain for a specific economic purpose, but which, because of ignorance of the risk involved, blissfully located in the green valley. It must be acknowledged, however, that all investments under our free enterprise system acknowledge and allow successfully for various risks, and it is submitted that in the final analysis either inability or refusal to evaluate the risk is responsible for all unwise encroachment. Thus, there is presented the need for educating John Q. Public on the virtues of determining his calculated risk.

Engineers and laymen confronted with flood plain losses can agree on a common objective, namely—"The reduction of flood damages by more intelligent use of the flood plains." In the paper just presented it is implied that one way of attaining this objective is through the utilization of land use regulations. It remains to be seen whether engineers, the courts, and the public will be willing to adopt a regulatory type approach. Four reasons were presented why widescale adoption of regulatory procedures has not been realized. Those reasons were briefly (1) lack of hydraulic data, (2) lack of land use

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data, (3) objections by property owners, and (4) lack of legal precedent. It is submitted that these four points again illustrate the present inability of individuals, courts, developers, and others to evaluate properly the risk involved in certain flood plain development.

A secondary primary problem to be considered is tightly interwoven among the various disciplines. Simply stated the problem becomes one of defining what is the right of the public and what is the right of the individual. Primary arguments for public control hinge on protection of the general welfare from the direct adverse physical effect of unwise development, and the secondary effect of modern floods on the community and the regional economy and public health. Countering these public needs is the desire for preserving greatest possible individual freedoms. The only adverse effects sustained from many encroachments is the damage to the facility itself. If that same facility can be shown to represent a long term economic gain can it be classified as a non-conforming use? Further, floods come in many shapes and sizes—there is the extraordinary and the routine. Does the same public right hold for all possible areas of inundation? The answer to these and other questions go far beyond the scope of this discussion. They will not be answered succinctly or quickly. Their answers will, at best, evolve with time for they are simply a portion of a more diverse question of local, state, and national policy. A question which the Engineers Joint Council recognized when they stated "The application of desirable water policies can only be carried out . . . . when an overall policy of land utilization is available."(1) Paradoxically, development of such a policy for flood plain areas must await a reasonable attainment in public understanding of the problem.

The foregoing comments implying the need for an educational approach to the problem of flood plain use have not been asserted in opposition to the author's implied plea for positive zoning procedures. However, the educational approach has been grossly overlooked at the local, state, and national level. Presumably the responsibility for development of adequate educational program could be a proper function of government.

The writer once suggested the desirability of interpreting and presenting available hydraulic data in the form of river plan and profile sheets.(2) Adequate strip maps can be developed in many parts of the country from available aerial photos. Sufficient information such as river mileages, section centers, and major highways would have to be identified to allow correlation of plan and profile. The profile sheets might well include a low water profile, profiles for floods of various percent chance of occurrence, and a profile for a so-called provisional or project design flood. Seldom could a stream be found where the available data seemed sufficient in every detail. However, treated as a design problem the task would reduce to certain standardizing procedures. For example, recent utilization of regional analyses of stream flow data could be a useful tool for such work.(3) It seems apparent that such data would be in much demand once investment houses and others realized the significance of the message it contains. Particularly would this be true where those who seek flood plain development are required to pay their proportionate share, based on benefits received, of the cost of future protection works.

Finally one cannot lose sight of the importance of the author's last statement. Bigger floods are yet to come. Zoning or less formal approaches to land use regulation can only be effective up to a certain limit. Thus land use regulations cannot be advocated as a panacea or complete cure, they simply

represent another tool for minimizing flood damage. If the thoughts expressed in the paper are sufficient to reawaken in the minds of only a few the possibility of utilizing that tool, the paper will have served its purpose.

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COAGULATION AND SEDIMENTATION[a]

J. M. DallaValle[b]
(Proc. Paper 1052)

SUMMARY

By assuming initially a monodisperse system of particles or nuclei, the chance of collisions leading to 1, 2, 3, etc. aggregated particles is computed according to the procedure used by Smoluchowski. The effect of settling is ignored by assuming the process is very fast. A discussion is included of the general law of settling of dilute suspensions as well as highly concentrated suspensions.

The breakup of matter into finer and finer particles can proceed only to a certain point before the phenomena of coagulation or coalesence begin to bring them together again. It may well be that the forces tending to disintegrate matter are eventually counterbalanced by others which act to bring particles together. In industry, fine particles are made to join together by a form of coagulation called polymerization and this is the basis of plastics manufacture, rubber synthesis and many other products. Often it is a transitory phenomena on the way to making a final product and thus receives little attention. Or, as in the case of water purification, it is a carefully controlled phenomena. Coagulation is not a phenomena characteristic of liquids alone. The manufacture of metallic oxides by burning pure metals exhibits this phenomena. In fact, with many metallic oxides, particle size control is achieved by letting the particles "grow"—in other words, to aggregate or coagulate while still in suspension.**

The tendency of particles to coalesce or coagulate is far more widespread than supposed. There can be no question but they act in all fine particle

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** We shall use the terms coagulation, coalesence, aggregation, cohesion and accretion synonymously and understand that they apply to the growth of particles by accumulation to their surfaces of other particles of the same species.

suspensions and are essential to the formation of stable sedimentary deposits. The constant stirring up of tidal basins, channels and shorelines by wave motion, of seasonal upheavals of lake bottoms by thermal inversions and even atmospheric dusts as they are swept about by winds would make this a difficult world in which to live in the absence of coagulation.

This paper proposes to outline the mechanism of coagulation and the binding forces between particles once they stick together, and then to show how particles settle to form the tremendous sedimentary deposits which constitute so large a part of the earth's crust.

Smoluchowski's Theory of Coagulation

It is noteworthy that since the fundamental investigations of Hardy (1900), Wiegner (1911), Smoluchowski (1918) and more recently, Muller (1926, 1928), Tuorila (1927) and Fuchs (1934), the basic theory of coagulation has been only slightly modified. It is especially to Smoluchowski that we owe our present appraisal of the mechanism of coagulation. Muller and Fuchs have extended certain aspects of Smoluchowski's theory, but for all its simplifications rectified in many respects by later investigators, Smoluchowski's basic concepts remain unchanged. His theory is applicable to all fluids—liquids and gases—and his equations have more universal applications with regard to coagulation phenomena than any of those derived with greater attention to detail. Considering the many extraneous complications affecting coagulations such as (in liquids) the electrolyte atmosphere and accompanying double-layer effects, and (in gases) the existance of electrostatic charges, it is indeed remarkable that Smoluchowski was able to develop so successful a theory; for these factors affect the mathematical treatment so as to render a solution of the problem impossible.

Smoluchowski used a very simple model for deriving his coagulation equations. He assumed that the particles were initially uniform in size and randomly distributed. He regarded them as subject to Brownian motion during the whole period of coagulation. He assumed that there were collisions between the particles as a result of their random motions and that on collision the particles formed a single particle. Rapid coagulation was regarded as a process in which particles on every contact adhered to each other on every contact; slow coagulation was attributed to the fact that only a fraction of the particles adhered on collision. We shall be concerned only with the first of these two types of coagulation in this paper. Why particles should adhere to each other on first contact in some instances and not in others will be discussed briefly later.

As indicated or implied above, coagulation is that process which leads to the growth of particles in a fluid atmosphere by accretion of other particles. If initially there are n_0 particles per unit volume and no settling occurs, at a time t after coagulation begins, there will be no change in the mass of particles suspended, but there will be some kind of a distribution consisting of single particles, double particles, triple particles, etc. If the particles remain small even after the coagulation process has proceeded for some time, or if the process is very rapid, we may ignore the effect of gravity. However, with time, all particles come under the influence of the gravitational field and the suspension will no longer consist of randomly distributed particles. In general, if the particle diameter is a function of time, $D(t)$, the

density of the suspension at any point z (from the bottom of the container) and time t is, after coagulation has ceased

$$\phi(z,t) = \rho_o + C_o\,[(\rho_s - \rho_o)/\rho_s] \int_0^{D_e} F(D_e)\,d(D_e) \qquad (1)$$

where ρ_o and ρ_s are the densities of the fluid and particle, respectively, Co is the concentration of the suspension at the moment coagulation ceases, D_e is the effective Stokes diameter and $F(D_e)$ its size distribution. In using Eq. (1), we may take $t = 0$ at the instant the rapid coagulation process is completed and the particles begin to settle. Both ϕ and C can, of course, be readily expressed on a number basis when it is convenient to do so.

We shall let n denote the number of particles in a unit volume at time t after coagulation has started. The reciprocal of this number, $1/n$, can be regarded as the average space occupied by a particle. If we have some method of measuring the change of n with time, we shall find that a plot of $1/n$ vs t yields a straight line with a positive slope. Experimental methods are available for determining n from instant to instant in a coagulating system and for both sols and aerosols, provided gravity has not affected the randomness of the distribution, data have been obtained which yield the simple but important equation

$$(1/n) - (1/n_o) = Kt \qquad (2)$$

in which K is called the "coagulation constant." Since the n's are usually quite large, it follows that the constant K for a given system is very small. The constant K will, of course, depend on temperature and whether the system is quiescent or turbulent.

Actually, Smoluchowski derived Eq. (2) using the following considerations: First he regarded the system to be initially monodisperse and randomly distributed and consisting of an average of n_o particles per unit volume. The volume of all the solids together was assumed negligible in comparison with the volume of the liquid. Next, with time increasing and with Brownian motion, all or a stated fraction of particles on collision adhere to each other. In order to calculate the number of particles colliding with a particle (assumed fixed), we may consider the particle to be surrounded by an imaginary sphere of radius r_o and apply the Fick diffusion equation giving the number of particles n' diffusing through a surface. This is a procedure for a steady state, but it can be shown to be sufficiently accurate. The Fick equation is

$$n' = 4\pi r^2 D(\partial n/\partial r)$$

where D is the diffusion coefficient. We can integrate this equation subject to the conditions:

$$n = n_o \quad \text{for} \quad r = \infty$$
$$n = 0 \quad \text{for} \quad r = r_o$$

The last condition states that the particle concentration at r_o is kept at zero, that is, the particle adheres. Integrating, subject to the conditions stated, the number of particles intercepted by the sphere is

$$n' = 4\pi r_o n D \qquad (3)$$

By way of simplification, we assume r_o and D to be the same for all particles. If each particle acts as a center at which coagulation takes place within a sphere of influence of radius r_o, then

$$(1/2)\, 4\pi r_o n^2 D = 2\pi r_o n^2 D$$

since two particles are involved to make a single particle. It follows from Eq. (3) that

$$n' = \frac{dn}{dt} = -\,2\pi r_o n^2 D \qquad (4)$$

an equation which is integrable if r_o and D can be shown independent of time. At this point, Smoluchowski proceeds along lines well reasoned in the kinetic theory of gases. Since diffusivity is an additive property, and since, because of size differences as coagulation proceeds, we have in the case of two particles, diffusivities D_1 and D_2, then we may write

$$D = D_1 + D_2$$

Also, on the basis of kinetic theory of gases, the new sphere of influence of two coagulated particles will be

$$r_o = (1/2)\,(r_{o1} + r_{o2})$$

in which r_{o1} and r_{o2} are the radii of influence of the two particles before coagulation. We note that if r_{o1} and r_{o2} are equal to twice the radii of the particles, the particles touch each other sufficiently close to pull together and coagulate. If the radii of the sphere of influence are greater than twice the radii of the particles' field of influence, then obviously the particles will always coagulate. With these assumptions

$$-\frac{dn}{dt} = \pi(D_1 + D_2)(r_{o1} + r_{o2})n^2 \qquad (5)$$

Einstein has shown that for particles in Brownian motion, the diffusion coefficient is

$$D = RT\bar{u}/N$$

where R is the gas constant, T the absolute temperature, N is Avogadro's number and $\bar{u}$ the mobility defined by Stokes law of resistance

$$\bar{u} = 1/3\pi\mu D_e$$

μ being the viscosity of the fluid. Thus,

$$D = (RT/N)\,(1/3\pi\mu D_e)$$

nd from Eq. (5), if D_{e1} and D_{e2} are the actual effective diameters of the oagulating particles, we get

$$-\frac{dn}{dt} = (RT/3\mu N)\,(D_{e1}^{-1} + D_{e2}^{-1})\,(r_{o1} + r_{o2})n^2$$

Now again, as a further simplification, we may say that the radii of influence re as their respective diameters—that is, $r_{o1} = aD_{e1}$ and $r_{o2} = aD_{e2}$, a eing a constant of proportionality. The last equation then becomes

$$-\frac{dn}{dt} = (RTa/3\mu N)\,(r_{o1} + r_{o2})^2\, n^2/r_{o1}\, r_{o2} \tag{6}$$

the sol or suspension is initially regarded as monodisperse and if as time roceeds the differences in sizes is not very great, then we may put $r_{o1} =$ $_{o2}$ = etc., and we obtain an expression which is independent of particle size

$$(1/n) - (1/n_o) = (2a/3)\,(RT/\mu N)\cdot t = Kt \tag{7}$$

$$K = (2/3)\,(RTa/\mu N) \tag{8}$$

here the limits of integration are obviously $n = n_o$ when $t = 0$ and $n = n$ hen $t = t$. Equation (7) is identical to Eq. (2). However, it includes the quation of state and an unknown constant a which must be evaluated by exeriment. It is indeed curious that Eq. (7) with all the simplifications and ssumptions made in deriving it should hold even for the early stages of oagulation. But there is much evidence that it applies very well to many oagulating systems until gravity begins to take over.

Muller (1926, 1928) has re-examined the general theory and has shown ow it can be applied to a system consisting of spherical particles of two diferent sizes. He has also indicated the modifications needed for dealing with oagulation of needle-like particles. Muller's treatment of these special ases are quite involved.

Fuchs (1934) appears to have been the last investigator to tackle the theretical aspects of coagulation. However, he dealt with aerosols and by inoducing concepts well-known in quantum theory advanced our knowledge of oagulation a great deal. His results, however, while more precise, remain be tested.

Equation (7) can also be applied to aerosols and this has been done with onsiderable thoroughness by Whytlaw-Gray (1932). For such applications, is necessary to introduce a correction in mobility $\bar{u}$ as defined by Stokes quation, which takes account of particle slippage between gaseous molecules. this case

$$\bar{u} = [1 + (2A\lambda/D_e)]/3\mu D_e$$

where λ is the mean free path of the gas molecules, A is a parameter depending on the ratio λ / De and other terms are as previously defined. The correction applied above is known as Cunningham's correction and a discussion of its significance as well as values of A are contained in most texts on physical or colloid chemistry.

Forces of Attraction

Any discussion of the forces tending to hold particles together leads to the very nature of matter itself. The following discussion is intended to elucidate on mechanism. It is necessarily brief and intended only to give an idea of the magnitude of the binding forces.

Let us assume that a particle in a coagulating system is momentarily fixed and is struck by another particle which has a kinetic energy of the order of 10^{-8} ergs. This is a very small amount of energy easily acquired by a particle 5 microns in size moving at a speed of about 10 cm per sec. But this amount of energy, small as it is, when given up in an inelastic collision with a cube of nickle 100 Angstroms on edge is sufficient to heat this cube to a temperature of about 100 deg C. (in air). Thus, if this energy were possessed by a particle of 100 Angstroms per edge and it struck a larger particle inelastically, then it would appear that there is sufficient energy to cause the two particles to coalesce. Collisions in such instances "fuse" the particles together.

But there are other forces of cohesion when particles come in contact that are referred to as van der Waal forces. These are forces exerted by unsatisfied atoms (as might occur at surface discontinuities of a freshly made crystal) and which on contact with similar atoms in another particle tend to bind each other together. These binding forces are of great intensity and are of the order of several tons per square inch. Thus, the probability of very small particles joining together on contact is indeed very great.

Particle Distribution Resulting from the Coagulation Process

As coagulation proceeds, single particles will collide to form doublets. Single particles will collide with doublets to produce triplets, etc. Again doublets will be colliding with doublets to form quadruplets, or with triplets to form quintuplets, etc. Since Eq. (7) gives the law of diminution of particle numbers as coagulation proceeds, it is of interest to see whether it is possible to determine the number of singletons, doublets, triplets, etc. present in a suspension at any time t after the process has begun. In this regard, we may again follow Smoluchowski. Let n_0 as before be the number of particles initially present in a unit volume of the suspension, and n the number of particles present at time t and composed of

n_1 particles which are singles
n_2 particles which are doublets
n_3 particles which are triplets
etc.
n_i particles which are i-lets

$$n = \sum_i n_i$$

In deriving Eq. (7), we have assumed that the coagulation constant K applies to all the particles. If each collision of two unit particles decreases the number of units by 2, while collisions with other sizes diminishes the number by one, then

$$-dn_1/dt = 2Kn_1^2 + 2Kn_1n_2 + 2Kn_1n_3 + \ldots = 2Kn_1n$$

For the decrease of doublets formed by coagulation of 2 unit particles

$$-dn_2/dt = -Kn_1^2 + 2Kn_2n_1 + 2Kn_2^2 + 2Kn_2n_3 + \ldots = -2Kn_1^2 + 2Kn_2n$$

and for triplets

$$-dn_3/dt = -2Kn_1n_2 + 2Kn_3n_1 + 2Kn_3n_2 + 2Kn_3^2 + 2Kn_3n_4 + \ldots$$

$$= -2Kn_1n_2 + 2Kn_3n$$

Proceeding in this manner, we then obtain a series of expressions for i-particles. Since the rate of decrease of all the particles is controlled by eq. (7), we may use this equation to eliminate time

$$- dn/dt = - \sum_i dn_i/dt$$

and thus obtain differential equations with $n_1, n_2, \ldots, n_i$ as variables. These equations are easily integrated and we obtain the geometrical sequence

$$n_1 = n^2/n_o$$

$$n_2 = n^2 (n_o - n)/n_o^2$$

$$n_3 = n^2 (n_o - n)^2/n_o^3$$

$$\ldots$$

$$n_i = n^2(n_o - n)^{i-1}/n_o^i \qquad (9)$$

These equations, when added, form a geometric series with a common term-to-term ratio of $(n_o - n)/n_o$. These are the most convenient form to work with, although (following Smoluchowski) the following arrangement for the general term is also useful:

$$n_i = n_o (t/t_o)^{i-1}/[1 + (t/t_o)]^{i+1}$$

where we define

$$t_o = 3\mu N/4RTn_o$$

which is of course, the reciprocal of K with a = 2, a generally accepted

value of this constant. Figure 1 shows how the ratio n/n_o changes with the ratio t/t_o with different values of n_i as a parameter. We see, therefore, that if we have a means of determining the number concentration of particles at any time t and the initial concentration when $t = 0$, then the distribution of coagulated particles in terms of singletons, doublets, triplets, etc. is readily determined.

We note in passing that the most frequent number of coagulated particles is given by the expression

$$\bar{N} = (2n_o - n)/n \tag{10}$$

or, in terms of the coagulation constant K and the time t

$$\bar{N} = 2Kn_o t + 1 \tag{11}$$

Note also as shown in Figure 1 that the maximum of each grouping shifts toward the right as time increases.

Sedimentation

When the effect of gravity takes over, the coagulated particles will settle—provided there is no further accretions—in accordance with well defined laws. At the instant gravity may be said to take over, the density of the suspension at any point z measured vertically in the suspension and time t is controlled by Eq. (1).*

Free Settling

We shall now recognize that there are two kinds of settling: (1) that form in which the particles in the act of settling do not interfere with each others motion. This is usually the case when the volume concentration of the suspension is less than five per cent of the total volume. (2) Hindered settling which occurs with thick suspensions and is such that the particles settle as a mass with an interface between clear fluid and suspension.

In free settling, the downward velocity of each particle is in accordance with Stokes law

$$u_s = (\rho_s - \rho_o)gD_e^2/18\mu = z/t \tag{12}$$

where z is the distance moved by the particle without acceleration in the time t and other terms have already been defined. If now in Eq. (1) z is fixed, we obtain by differentiation of and application of Eq. (12) that

$$\frac{d\Phi}{dt} = - \frac{D_e}{2t} F(D_e) \tag{13}$$

in which Φ is the ratio of concentration of particles at z at time t and the

* Throughout this section time $t = 0$ is taken at the moment coagulation ceases and gravity takes over.

oncentration when the particles were uniformly distributed in the suspen-
ion (time, $t = 0$), that is, $C(t)/C_o = \phi$. If, on the other hand t is constant
nd we wish to obtain the distribution at various levels in the suspension

$$\frac{dZ}{dz} = -\frac{D_e}{2z} F(D_e) \tag{14}$$

nd in this case, Z refers to the ratio of concentrations at various z-levels
the initial concentration, $C(z)/C_o$. Equations (13) and (14) serve to give a
omplete history of the free-settling phenomena.

indered Settling

In the case of hindered settling, the interface between the liquid and the
uspension moves at a constant velocity u_o. This problem may be handled
much the same was as flow through a packed bed of particles with the par-
cles moving instead of the liquid. In fact, the Kozeny-Fair-Hatch equation
the form proposed by L. Hatch (1943) may be used. This equation applied
an expanded bed of fine particles with a porosity ϵ. The form we use
ere applies to slow settling, but Hatch indicates well the general form of the
quation for other types of flow. The equation is

$$(u_o^2/gD_s)\,[(1-\epsilon)/\epsilon^3] = 5.56 \times 10^{-3}(\rho_s - \rho_o)\,u_o D_s/\mu \tag{15}$$

here D_s is the specific surface diameter obtained by permeametric meth-
ds (DallaValle, 1948). Other terms in the equation are defined elsewhere in
is paper. Note that as Eq. (15) is written that the left hand portion is a
odified Froude group and the right hand side a modified Reynolds group.
ence in general, we may expect the following equation to govern hindered
ettling:

$$Fr_m = \phi\,(Re)_m \tag{16}$$

his equation holds very well up to porosities of 0.85 and for particles rang-
g from 5 to more than 1000 microns in water, oil or glycol suspending
edia. In fact, it is a convenient method for measuring the characteristic
ameter D_s of particles. However, with coagulated or aggregated particles
hich in settling carry along with them an adhered layer of fluid, density cor-
ctions are necessary. In order to correct for this adhered liquid, it is
cessary to follow along the lines indicated by Steinour (1944).

SUMMARY

By assuming initially a monodisperse system of particles or nuclei, the
ance of collisions leading to 1, 2, 3, etc. aggregated particles is computed
cording to the procedure used by Smoluchowski. The effect of gravitation
ignored by assuming the process is very fast. A discussion is included of
e general law of settling of dilute suspensions as well as highly concen-
ated suspensions.

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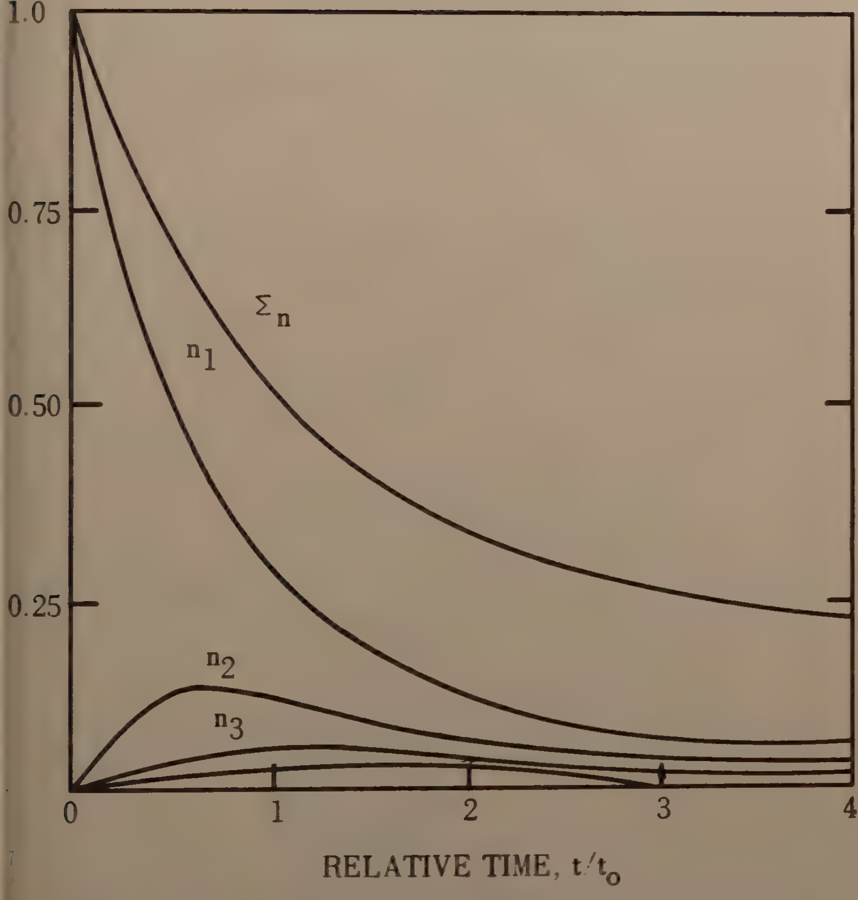

Figure 1.

Relation between number of particles and time in rapid coagulation.

DIVISION ACTIVITIES
HYDRAULICS DIVISION
Proceedings of the American Society of Civil Engineers

NEWS BULLETIN-August 1956

The April issue listed the committees for the Society business year 1955-1956 and referred to the Official Register for the names of members serving on Task Forces. The following additions and corrections should be made in your Register to make the record of those at work in the Division complete:

Task Force On Energy Dissipators for Spillways and Outlet Works:
Add Rex Elder

Task Force on Hydraulic Aspects of the Design and Operation of Spillways and Crest Gates:
Alvin J. Peterka is now chairman
Marvin J. Webster is now a member
Messrs. Nalder and Brown resigned.

Task Force on Nomenclature for Hydraulics: (Note the change in the name of this task force)
C. A. Betts is also on this task force

Task Force on Water Measurements:
Add A. L. Jorissen and C. A. Wright

Task Force on Sediment Distribution in Reservoirs:
Samuel Shulits, chairman
Paul C. Benedict
Lloyd C. Fowler
Carl R. Miller
John B. Stall

The Hydrology Committee has established a Task Force on Hydrologic Data. The Task Force is staffed by the following members:
D. W. Van Tuyl, chairman

C. O. Clark	W. W. Wheeler	Victor A. Koelzer
L. L. Harold	Robert R. Balmer	Edward J. Schaefer
H. E. Hudson	Verne Alexander	Clayton H. Hardison
		Walter G. Schulz

C. C. McDonald, Coordinator

A Task Force on Spillway Design Floods has been authorized.
Harvey O. Banks is chairman

Note: No. 1956-18 is part of the copyrighted Journal of the Hydraulics Division of the American Society of Civil Engineers, Vol. 82, HY 4, August, 1956.

The Committee on Research has two task forces at work. The Task Force on Aerated Flow in Open Channels consists of:

Warren W. De Lapp, chairman
William J. Bauer
Donald Colgate
Joseph P. Lawrence
(Frank B. Campbell is Contact Member)

The Task Force on Cavitation in Hydraulic Structures consists of:

F. R. Brown, chairman
R. A. Elder
V. E. Johnson, Jr.
M. B. McPherson
A. J. Peterka
(J. M. Robertson is Contact Member)

Mr. Thornton J. Corwin, Jr., past chairman of the Executive Committee, recently resigned from the committee for reasons of health. It is regretted exceedingly that illness has made it necessary for him to give up his work in the Division. A recent note from Thor indicates that he is making a satisfactory recovery and is now able to be at his office in the Pacific Gas and Electric headquarters.

Dr. Arthur T. Ippen has been appointed by the ASCE Board of Direction as a member of the Executive Committee of the Hydraulics Division. This appointment is effective immediately for a term ending in October of 1960. Art brings with him a wealth of experience in hydraulics and committee work from which the Division will undoubtedly profit.

Committee and Task Force Meetings

The following meetings have been held so far this year since October 1955

Executive Committee	Denver	November 7-8, 1955
Committee on Tidal Hydraulics	Washington, D. C.	January 17, 1956
Committee on Hydrology	Dallas, Tex.	February 13-14, 1956
Committee on Design	Knoxville, Tenn.	May 3-4, 1956
Committee on Sedimentation	"	June 4-5, 1956
Task Force on Sediment Distribution in Reservoirs	Urbana, Ill.	May 15-17, 1956

The Committee on Research will hold its next meeting at Chicago, August 25, 1956

The Executive Committee will hold its next meeting at Madison, Wisconsin, August 21, 1956

HYDRAULICS DIVISION TO HAVE BIG PROGRAM AT PITTSBURGH CONVENTION October 15-19, 1956

Place: William Penn Hotel

Technical Sessions have been formulated by the following committees:

Design	1 session
Floods	1 session on flood insurance
Research	1 session on turbulence in fluid flow
	1 session on cavitation in hydraulic structures
	1 general session
Tidal Hydraulics	2 sessions sponsored jointly by the Committee on Tidal Hydraulics and the Committee on Coastal Engineering of the Waterways Division

Committeemen and Task Force chairmen are invited to use the Hydraulics Division News Bulletin to:

a) Insert outline of proposed work and invite suggestions thereon

b) Insert resume of work or report and invite suggestions thereon

c) Insert questions on specific problems on which information is required

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Committee on Publications

Chairman H. G. Dewey reports that since the publication of JOURNAL No. HY 2, the Division has passed on 14 papers to the Manager of Technical Publications for publication in future issues of the JOURNAL; eight (8) papers are being revised by the authors; and three (3) papers are under active review. This indicates another very active year in publication of Hydraulic papers.

HYDRAULICS DIVISION MEMBERSHIP Questionnaire

It has been several years since the membership of the Hydraulics Division has been canvassed to determine the interest and availability of the individual members for service in Division activities. All who are interested in participation in Division activities are urged to fill out the following questionnaire and return to the Secretary of the Division.

QUESTIONNAIRE OF PARTICIPATION OF HYDRAULICS DIVISION

Name ______________________________

Address ______________________________

Position

Consultant ____
Manufact'r ____
Government ____
School ____
Private ____
Other ____
Retired ____

Field of Interest

Design ____
Hydrology ____
Research ____
Ind. Hydr. ____
Other ____

Preparation of Technical Paper

By self ____
With other ____

Edit Papers

Yes ____
No ____

Division Committee Service

Design ____
Hydrology ____
Research ____
Publications ____
Floods ____
Tidal Hydraulics ____
Sedimentation ____
Stevens or Hilgard Prize ____
Other Member ____

Review Papers

Yes ____
No ____
Floods ____
Sedimentation ____
Gr'd Water ____
Snow, Ice and Permafrost ____
Design ____
Hydrology ____

Research General ____
Research Specialized (Indicate specialty) ____

Comments and suggestions ______________________________

____

Subjects recommended for discussion at meetings ______________________________

____

____

Suggested Division meeting place ______________________________

Please return to Harold M. Martin, Secretary, Hydraulics Division, ASCE, Bureau of Reclamation, Denver Federal Center, Building 56, Denver 2, Colorado.

PROCEEDINGS PAPERS

[The] technical papers published in the past year are identified by number below. Technical-[divis]ion sponsorship is indicated by an abbreviation at the end of each Paper Number, the [symb]ols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering [Mech]anics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), [Sanit]ary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and [Mapp]ing (SU), and Waterways and Harbors (WW) divisions. Papers sponsored by the Board of [Dir]ection are identified by the symbols (BD). For titles and order coupons, refer to the appro[priat]e issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were [publi]shed in Journals of the various Technical Divisions. To locate papers in the Journals, the [symb]ols after the paper numbers are followed by a numeral designating the issue of a particular [Journ]al in which the paper appeared. For example, Paper 861 is identified as 861 (SM1) which [indic]ates that the paper is contained in issue 1 of the Journal of the Soil Mechanics and Founda[tions] Division.

VOLUME 81 (1955)

[AUG]UST: 761(BD), 762(ST), 763(ST), 764(ST), 765(ST)^c, 766(CP), 767(CP), 768(CP), 769(CP), [77]0(CP), 771(EM), 772(EM), 773(SA), 774(EM), 775(EM), 776(EM)^c, 777(AT), 778(AT), [77]9(SA), 780(SA), 781(SA), 782(SA)^c, 783(HW), 784(HW), 785(CP), 786(ST).

[SEP]TEMBER: 787(PO), 788(IR), 789(HY), 790(HY), 791(HY), 792(HY), 793(HY), 794(HY)^c, [79]5(EM), 796(EM), 797(EM), 798(EM), 799(EM)^c, 800(WW), 801(WW), 802(WW), 803(WW), [80]4(WW), 805(WW), 806(HY), 807(PO)^c, 808(IR)^c.

[OCT]OBER: 809 (ST), 810 (HW)^c, 811 (ST), 812 (ST)^c, 813 (ST)^c, 814(EM), 815(EM), 816(EM), [81]7(EM), 818(EM), 819(EM)^c, 820(SA), 821(SA), 822(SA)^c, 823(HW), 824(HW).

[NOV]EMBER: 825(ST), 826(HY), 827(ST), 828(ST), 829(ST), 830(ST), 831(ST)^c, 832(CP), [8]33(CP), 834(CP), 835(CP)^c, 836(HY), 837(HY), 838(HY), 839(HY), 840(HY), 841(HY)^c.

[DEC]EMBER: 842(SM), 843(SM)^c, 844(SU), 845(SU)^c, 846(SA), 847(SA), 848(SA)^c, 849(ST)^c, [8]50(ST), 851(ST), 852(ST), 853(ST), 854(CO), 855(CO), 856(CO)^c, 857(SU), 858(BD), 859(BD), [8]60(BD).

VOLUME 82 (1956)

[JAN]UARY: 861(SM1), 862(SM1), 863(EM1), 864(SM1), 865(SM1), 866(SM1), 867(SM1), 868(HW1), [8]69(ST1), 870(EM1), 871(HW1), 872(HW1), 873(HW1), 874(HW1), 875(HW1), 876(EM1)^c, 877 [(H]W1)^c, 878(ST1)^c.

[FEB]RUARY: 879(CP1), 880(HY1), 881(HY1)^c, 882(HY1), 883(HY1), 884(IR1), 885(SA1), 886(CP1), [88]7(SA1), 888(SA1), 889(SA1), 890(SA1), 891(SA1), 892(SA1), 893(CP1), 894(CP1), 895(PO1) [89]6(PO1), 897(PO1), 898(PO1), 899(PO1), 900(PO1), 901(PO1), 902(AT1)^c, 903(IR1)^c, 904 [(P]O1)^c, 905(SA1)^c.

[MAR]CH: 906(WW1), 907(WW1), 908(WW1), 909(WW1), 910(WW1), 911(WW1), 912(WW1), 913 [(W]W1)^c, 914(ST2), 915(ST2), 916(ST2), 917(ST2), 918(ST2), 919(ST2), 920(ST2), 921(SU1), [9]22(SU1), 923(SU1), 924(ST2)^c.

[APR]IL: 925(WW2), 926(WW2), 927(WW2), 928(SA2), 929(SA2), 930(SA2), 931(SA2), 932(SA2)^c, [9]33(SM2), 934(SM2), 935(WW2), 936(WW2), 937(WW2), 938(WW2), 939(WW2), 940(SM2), 941 [(S]M2), 942(SM2)^c, 943(EM2), 944(EM2), 945(EM2), 946(EM2)^c, 947(PO2), 948(PO2), 949(PO2), [9]50(PO2), 951(PO2), 952(PO2)^c, 953(HY2), 954(HY2), 955(HY2)^c, 956(HY2), 957(HY2), 958 [(S]A2), 959(PO2), 960(PO2).

[MAY]: 961(IR2), 962(IR2), 963(CP2), 964(CP2), 965(WW3), 966(WW3), 967(WW3), 968(WW3), 969 [(W]W3), 970(ST3), 971(ST3), 972(ST3)^c, 973(ST3), 974(ST3), 975(WW3), 976(WW3), 977(IR2), [97]8(AT2), 979(AT2), 980(AT2), 981(IR2), 982(IR2)^c, 983(HW2), 984(HW2), 985(HW2)^c, 986(ST3), [98]7(AT2), 988(CP2), 989(AT2).

[JUN]E: 990(PO3), 991(PO3), 992(PO3), 993(PO3), 994(PO3), 995(PO3), 996(PO3), 997(PO3), 998 [(S]A3), 999(SA3), 1000(SA3), 1001(SA3), 1002(SA3), 1003(SA3)^c, 1004(HY3), 1005(HY3), 1006 [(H]Y3), 1007(HY3), 1008 (HY3), 1009 (HY3), 1010 (HY3)^c, 1011 (PO3)^c, 1012 (SA3), 1013 (SA3), [10]14(SA3), 1015(HY3), 1016(SA3), 1017(PO3), 1018(PO3).

[JUL]Y: 1019(ST4), 1020(ST4), 1021(ST4), 1022(ST4), 1023(ST4), 1024(ST4)^c, 1025(SM3), 1026 [(S]M3), 1027(SM3), 1028(SM3)^c, 1029(EM3), 1030(EM3), 1031(EM3), 1032(EM3), 1033(EM3)^c.

[AUG]UST: 1034(HY4), 1035(HY4), 1036(HY4), 1037(HY4), 1038(HY4), 1039(HY4), 1040(HY4), [10]41(HY4)^c, 1042(PO4), 1043(PO4), 1044(PO4), 1045(PO4), 1046(PO4)^c, 1047(SA4), 1048 [(S]A4)^c, 1049(SA4), 1050(SA4), 1051(SA4), 1052(HY4), 1053(SA4).

[c. D]iscussion of several papers, grouped by Divisions.

AMERICAN SOCIETY OF CIVIL ENGINEERS